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Sinan Salman[1] and Suzan Alaswad[2]

[1]College of Technological Innovation, Zayed University, Abu Dhabi, UAE
[2]College of Business, Zayed University, Abu Dhabi, UAE

Abstract

With the increasing global stock of vehicles, traffic congestion is becoming more severe and costly in many urban road networks. Road network modeling and optimization are essential tools in predicting traffic flow and reducing network congestion. Markov chains are remarkably capable in modeling complex, dynamic, and large-scale networks; Google's PageRank algorithm is a living proof. In this article, we leverage Markov chains theory and its powerful statistical analysis tools to model urban road networks and infer road network performance and traffic congestion patterns, and propose an optimization approach that is based on Genetic Algorithm to model network-wide optimization decisions. Such decisions target relief from traffic congestion arising from sudden network changes (e.g. rapid increase in vehicles flow, or lanes and roads closures). The proposed network optimization approach can be used in time-sensitive decision making situations such as crisis response management, where decision time requirements for finding optimal network design to handle such abrupt changes typically don't allow for the traditional agent-based simulation and iterative network design approaches. We detail the mathematical modeling and algorithmic optimization approach and present preliminary results from a sample application.

Keywords
Operations research, Markov chains, road network optimization, genetic algorithms

1. Introduction

Traffic congestion is growing fast in major cities around the world due to the increasing global stock of vehicles. Actually, it is projected that the global stock of vehicles will increase from 800 million vehicles in 2002 to over 2 billion vehicles by 2030 [1]. Often, choices made in developing transportation infrastructure have profound impacts on network performance. Transportation authorities typically employee a mixture of transportation network evaluation and design techniques to help it understand current and future challenges of this complex problem and produce transportation network solutions to meet such challenges. These approaches typically fall into one of two categories. First, traffic simulation which simulates existing or proposed road network traffic [2,3] with varying levels of granularity that enables different levels of accuracy in a trade off with computational cost. Second, static and dynamic traffic assignment models that are based on road network equilibrium approaches to estimate congestion and design road network solutions to minimize it [3,4]. Although both approaches can be very useful in certain applications, both suffer from shortcomings that prevent their use in other applications: 1) traffic simulation computational cost is typically very high such that real time analysis is not possible, 2) traffic assignment models utilize a unified driver behavior, i.e. based on user equilibrium, that assumes uniform driver's knowledge of the entire network performance which allow them to select routes accordingly.

In this article we aim to optimize the efficiency of existing transportation infrastructure by employing a recently developed road network evaluation approach, which is inspired by Google's PageRank algorithm and was applied to the traffic assignment problem to predict vehicles movement in urban road networks [5,6]. Our work builds on this evaluation approach and utilizes Genetic Algorithm (GA) to optimize a road network's traffic pattern through selectively converting two-way roads to one-way roads to arrive at an improved road network performance without requiring infrastructure network modification such as lane additions or road constructions. This approach can be utilized in time-sensitive decision making situations, such as crisis response management, when sudden and unpredicted traffic pattern changes take place in a network due to unforeseen events (such as localized disasters, or major accidents) rendering parts of the network heavily congested as a result of the event. Finding optimal network

design to handle such abrupt changes typically don't allow for the traditional agent-based simulation and iterative network design approaches. Although our approach utilizes GA, which typically is associated with long optimization runs, our experimental results demonstrate that our approach produces considerable network performance improvement for a real size city in a relatively short timeframe, allowing this approach to be deployed when such time-sensitive decision making is needed. This can be characterized as an intelligent road network traffic pattern design system, one which can analyze complex existing networks and suggest optimal solutions taking into consideration far reaching inter-dependencies of such changes on the entire network.

During the last few decades, urban network modeling has received great attention – see [7] for a comprehensive review on urban network models and several model classification schemes. One popular approach to model urban network is using flow models which are usually classified according to their representation granularity of traffic flow and behavior into three models: microscopic, macroscopic, and mesoscopic. Network equilibrium models are typically used to model and solve road/transportation Network Design Problem. Different approaches that generally utilize the bi-level user equilibrium approach were proposed in the literature to model and solve the Network Design. The Majority of these approaches are based on the assumption that road users' behavior can be predicted by User Equilibrium equations. For example, Zhang and Gao (2007) utilize a bi-level model, where the upper-level model minimizes total system cost and the lower-level model accomplishes user equilibrium assignment, to solve a proposed lane reallocation (during peak periods) problem. Long, et.al. (2010) proposed a turning restriction road network design problem and used bi-level optimization model to minimize total system travel time using branch and bound strategies. Long, et.al. (2014) extended the problem to a bi-objective, bi-level optimization problem where the total system travel time and total vehicle emissions are minimized using artificial bee colony algorithm (ABC).

Despite User Equilibrium popularity in literature, little effort has been expended to determine whether real world network flow patterns are accurately described by it [11]. Furthermore, models incorporating more realistic behavior principles suffer from solution methodology convergence issues [4], which complicates its application to realistic situations. Alternatively, the use of Markov chains to model road networks has become of interest due to their capability in modeling complex, dynamic, and large-scale networks. Although Markov chains use in transportation systems is diverse and numerous, its application to traffic assignment and road network design problems is just starting to gain attention recently. This line of research can be traced to earlier studies, e.g. [12-15], however, it was still a novel and not yet widely utilized approach until researchers started to apply inspirations from Google's PageRank algorithm to the traffic assignment problem. One of the early applications can be found in [5] which proposed the use of the PageRank algorithm to predict human movements in urban road networks. Nevertheless, this approach was first fully analyzed by Crisostomi et al. (2011) through a macroscopic model inspired by Google's PageRank algorithm. The authors model road networks as Markov chains applying its analytical tools to infer non-evident properties and compare its theoretical expectations to network characteristics obtained through simulation results, validating the new approach. It is noteworthy that the proposed model depends on easily accessible data such as: ratio of vehicles moving between road links on intersections, road link lengths, road link average vehicles speed, which are available through modern road network sensory infrastructures. Faizrahnemoon et al. (2015) applied the same Markov chain model proposed in [6] to multi modal public transportation networks, and proposed network design changes to improve the network performance. Reiter (2015) applied a similar Markov chains model to the greater Philadelphia region highway network by modeling links between highway exists as the Markov chain states and basing his probabilities on historical data. While previous studies utilized Markov chains in analyzing road networks, this work contribution is the expansion of Markov chain use from network evaluation into network optimization using GA as the optimization vehicle. GA has been used in User Equilibrium models to optimize road networks. For example, Jia, et al. (2009) utilize a bi-level model to optimize road network design through the selection between candidate links through the use of GA, simulated annealing (SA), and ABC algorithm. Sharma & Mathew (2011) presented a multiobjective road network optimization model which minimizes emissions and travel time and used GA as the to solve the network design problem. Szeto et al. (2014) proposed a network design optimization problem that considers emissions and noise costs, and solved the problem using chemical reaction optimization and GA. However, optimizing road network design using Markov chains while utilizing GA, to the best of our knowledge, has not been investigated yet.

The rest of this paper is organized as follows. First, we provide a brief primer on the use of Markov chains in road network evaluation. Next, we setup the problem, present the optimization model and GA approach. Then, we present experimental results on a real-size problem. Finally, we conclude the article and point to future work.

2. Traffic Evaluation Model

Our work utilizes the Markov chain road network evaluation model proposed by Crisostomi et al. (2011) which we summarize in this section. A Markov chain is a stochastic process characterized by the Markov property, which states that the future state depends only on the present state, or in other words, the probability of a random variable to be in a given state only depends on its previous state and not on the path it took to arrive at that previous state:

$$P(x_{k+1} = j \mid x_k = i) = P(x_k = j \mid x_{k-1} = i) = p_{ij} \tag{1}$$

Throughout this article we use discrete-time, finite-state, homogeneous Markov chains. A Markov chain is completely described by its transition matrix $P = [p_{ij}]$, where p_{ij} denotes the probability of moving from state i to state j. If a Markov chain is ergodic, the steady state probabilities π (aka stationary distribution) are given by $\pi P = \pi$. The vector π can also be calculated by finding P's eigenvector that is associated with the eigenvalue 1. Finally, the mean first passage time (from any state to any other state) for a Markov chain is known as the Kemeny constant, and can be calculated from the P eigenvalues ordered descending, such that $\lambda_1 = 1$ and $\lambda_i \leq 1$ for $i \neq 1$:

$$K = \sum_{j=2}^{n} \frac{1}{1 - \lambda_j}$$

$$\tag{2}$$

Since Markov chains are frequently used to describe networks, it is also customary to utilize graph theory to describe the same networks. Consider the directed weighted graph $G(V, E, P)$ with node set V, edge set E, and weight matrix tP, consisting of the turning probabilities tp_{ij} from one edge to another, where $0 \leq tp_{ij} \leq 1$ if $(i, j) \in E$ and $tp_{ij} = 0$ otherwise. Figure 1 shows two example intersections and some of their turning probabilities. Using this terminology, we can describe an entire city's road network as a directed network graph and model it using Markov chains to analyze network properties and performance indicators, such as the Kemeny constant which has been used in the literature to measure the mean expected time to arrive at the different states from an arbitrary state. We refer to edges and roads interchangeably going forward.

(a)   (b)

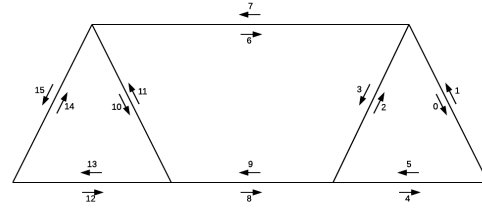Figure 1. Two example edge-to-edge turning probabilities.

Figure 2. A simple road network with 16 edges.

To illustrate the network evaluation method we consider the simple road network consisting of 16 edges $E = (0, 1, \dots, 15)$, as shown in Figure 2. Let all edges share the same average speed of 60 km/hr, and width of 2-lanes except for edges 6, 7, 8, and 9 which are single lane roads. Lastly, let all edges be of equal length of 1 km except for edges 6 and 7 which are of length 1.5 km. Let the turning probabilities tP be:

$$tP = \begin{bmatrix}
0 & 0.1 & 0 & 0 & 0 & 0.3 & 0.6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.1 & 0 & 0.9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.1 & 0.3 & 0 & 0 & 0 & 0.6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.9 & 0 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.3 & 0 & 0 & 0 & 0.1 & 0.6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.3 & 0.1 & 0 & 0 & 0 & 0.6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.1 & 0 & 0 & 0.6 & 0 & 0.3 & 0 & 0 & 0 \\
0 & 0.6 & 0 & 0 & 0 & 0.3 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.1 & 0 & 0 & 0 & 0.3 & 0.6 & 0 \\
0 & 0 & 0 & 0.6 & 0.3 & 0 & 0 & 0 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.1 & 0 & 0 & 0 & 0.9 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.6 & 0 & 0 & 0.1 & 0 & 0.3 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.6 & 0 & 0 & 0 & 0.1 & 0.3 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.6 & 0 & 0 & 0.3 & 0 & 0.1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.9 & 0 & 0 & 0 & 0.1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.6 & 0 & 0 & 0 & 0.3 & 0.1 & 0
\end{bmatrix}$$

Although the above turning probability matrix tP is also a Markov transition probability matrix, however it does not take into account that vehicles will take different times in traversing different roads due to differing road

characteristics, i.e. average speed and length. This can be incorporated into the model by calculating p_{ii}, which is the self-loop probability. By computing and normalizing travel times tt_i for all edges such that the smallest travel time is 1, we can compute the self-loop probability for all edges using eq.3, and the off-diagonal transition probabilities will be modified using eq.4.

$$p_{ii} = \frac{tt_i - 1}{tt_i}, \qquad i = 1, \dots, n \tag{3}$$

$$p_{ij} = (1 - p_{ii})\, tp_{ij}, \ i \neq j \tag{4}$$

The new Markov transition matrix P (referred to as transition matrix onwards), is then used to calculate π by finding P's eigenvector that is associated with the eigenvalue 1, as stated by the Perron-Frobenius theorem (Langville and Meyer 2006). Finally, given an estimated total number of vehicles V traversing the road network, and road length L_i and number of lanes N_i data, one can calculate road vehicle density (vehicles/km/lane) using eq.5, which enables granular evaluation of traffic conditions at each of the network roads. In particular, road traffic conditions can be compared to reference values published by the Highway Capacity Manual (HCM2010), shown in Table 1.

$$D_i = \frac{V \pi_i}{L_i N_i} \tag{5}$$

In conclusion of this section we list Figures 3(a through c) which illustrates the Markov chain evaluation model results for the simple road network example using a total of 100 vehicles on the network; Figure 3(a) shows stationary distribution probabilities for each of network roads, Figure 3(b) shoes traffic density after taking into consideration road length and number of lanes, and Figure 3(c) shows a heat map for simple network traffic density.

Table 1. Highway level of service (LOS) and congestion conditions, adapted from (HCM2010)

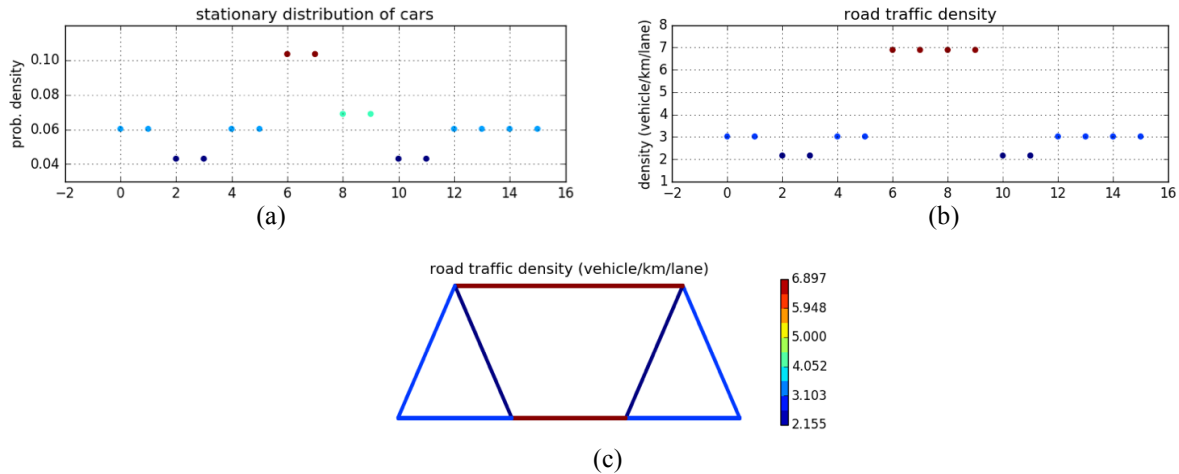| LOS	Traffic Conditions	Volume/Capacity Ratio	Maximum Density (vehicle/km/lane)
A	Free	0.35	7
B	Stable	0.54	11
C	Stable	0.77	16
D	High Density	0.93	22
E	Near Capacity	1.00	28
F	Breakdown	Unstable	>28

(a)

(b)

(c)

Figure 3. Markov chain evaluation model results for the simple road network consisting of 16 edges

3. Optimization Method

In heavily congested road networks, city planners and transportation authorities often turn to road direction conversion (two-way to one-way) for selected roads to enable larger traffic volume capacities and relieve congestions. Our approach is to design an optimization system that identifies direction conversion road candidates while evaluating the conversion total impact on the entire network, taking into consideration the interdependencies between different roads

in the network. In the case of an abrupt congestion resulting from a localized event, this approach can be used to identify quick, temporary, and comprehensive road direction conversion response plan to alleviate the congestion.

Therefore, the problem statement is: when a road network exhibits localized congestion due to unexpected traffic patterns, what is the set of roads that can be converted to one-way to help reduce the resulting congestion. Before defining our mathematical model, it will be beneficial to draw the reader's attention to the fact that rows (columns) in a transition matrix represent outbound (inbound) transition probabilities from (into) a road. Now, since congestion reduction is our focus, we formulate the model to minimize the maximum vehicle density in the road network, D_{max}:

$$\text{minimize } D_{max}$$
$$\text{s.t.:}$$
$$\frac{V\pi_i}{\widehat{L_i}N_i} \leq D_{max} \tag{6}$$
$$\pi\widehat{P} = \pi \tag{7}$$
$$\widehat{P} = H_d^{-1} P X_d \tag{8}$$
$$H_d = diag[\,(P X_d + O_d)\,\mathbb{1}_n\,] \tag{9}$$
$$O_d^{i,j} = \begin{cases} 1, & i = j, \; (P X_d \mathbb{1}_n)_i = 0 \\ 0, & otherwise \end{cases} \tag{10}$$
$$X_d^{i,j} = \begin{cases} 1, & i = j, and \; road \; i \; flow \; is \; unaltered \\ 0, & otherwise \end{cases} \tag{11}$$
$$X_d^{i,i} + X_d^{r(i),r(i)} > 0 \tag{12}$$
$$\widehat{L_i} = \left(2 - X_d^{r(i),r(i)}\right)L_i \tag{13}$$
$$r(i) = j \quad when \; i,j \; are \; a \; pair \; of \; opposing \; direction \; roads \tag{14}$$
$$\text{where: } i = 1, \dots, n \text{ and } j = 1, \dots, n$$

In the above model, V, L_i, N_i, P are estimated number of vehicles traversing the network at concurrently, road i length, and road i number of lanes, and network transition probability matrix, respectively, and are all collected data from the network under question. The diagonal decision variable matrix X_d designed to determine road flow restriction, such that if $X_d^{i,i} = 0$, then road i inbound flow is completely eliminated (achieved by post multiplying X_d with P in eq.8), and $X_d^{i,i} = 1$ when road i inbound flow is unaltered. This eliminates all vehicle traffic on the affected road allowing $\widehat{L_i}$ for the opposite direction road to take twice its normal value, and effectively doubling the road capacity in the opposite direction, achieving the goal of converting a road to a single-direction road with double the traffic volume capacity. $\widehat{P}$ is the modified transition probability matrix for the modified network as defined by the decision variable X_d, and π is the stationary distribution vector associated with $\widehat{P}$. The diagonal matrix H_d is a scaling matrix used to maintain transition probability proportions after the elimination of inbound transition probabilities after post multiplying X_d with P. The diagonal matrix O_d denotes absorbing states (roads); diagonal element $O_d^{ii} = 1$ when all transition probabilities for row i is zero, or $(P X \mathbb{1}_n)_i = 0$ after the elimination of inbound transition probabilities for road i. Vector $\widehat{L_i}$ is the new number of lanes based on the restriction of its opposite direction road. $\mathbb{1}_n$ is the column vector of all ones and size n. Finally, $diag[a]$ denotes the diagonal matrix generated by vector a. However, the model presents multiple challenges in regard to solution methodology using mathematical programming methods, this is due to the non-linear constraints eq.7 and eq.8, and the use of $diag[a]$ in eq.9 to create square diagonal matrices from vectors to achieve the scaling effect. These model shortcomings can be easily handled using GA.

GA is an optimization search heuristic characterized by the repetitive evolution of a population of candidate solutions to the underling problem. It achieves evolution through crossover and mutation operations that generate new offspring candidate solutions from the previous population generation to explore new areas of the solution space using the best characteristics of previous population generations. For the GA to work efficiently in exploring the solution space, an effective scheme for generating feasible network candidates is needed. For this approach, we selected a solution gene encoding such that each gene represent one of three states for every pair of opposing direction roads; the three states are: no change, the first road direction is reversed, and the second road direction is reversed. This solution encoding allows for the elimination of constraint eq.12 while generating feasible network candidate solutions, and at the same time reduce the number of genes by half rendering the solution space (3^n) considerably more manageable. In addition, GA typically is more efficient when solving unconstrained problems. Since the remaining constrains are all equality constraints, they can all be incorporated into the objective function calculation subroutine used in the GA computer code. This allows for the unconstrained optimization of the problem defined above using GA. Applying the GA

optimization approach to the simple network example discussed in section 3 proved to be trivial as the two best solutions of reversing links (6,9) or (7,8) were found in few generations with limited population size. This is expected as the solutions space ($3^8 = 6,561$) is not large.

4. Experimental Results

We present here experimental results from applying the proposed approach to a realistic size network, the island portion of the city of Abu Dhabi, UAE, and included 360 roads to model the main road arteries for the city, resulting in an impressive problem size ($3^{180} = 7.6 \times 10^{85}$). In this experiment an assumption of 10,000 vehicles traversing the network simultaneously was made, and turning probabilities were generally set in a similar fashion to Figure 1(a) unless certain turns are not present/allowed by the nature of a junction, in which case the remaining probabilities were scaled to ensure a total probability of 1.0. The following GA parameters were utilized in this problem: chromosome size (180), population size (200), total generations (1000), cross over probability (50%), mutation probability (30%), gene mutation probability (2%), and top 20 individual solutions were kept for the following generation.

Figure 5 summarize the GA run results in which a substantial improvement of 46.3% in maximum vehicle density was achieved; from 34.2 to 19.3 veh/km/lane. Although the best solution was only achieved at the later part of the optimization GA run, it is obvious that much of the gains in the objective function were achieved early on in the run (D_{max} = 21 by generation 97). For this example the network Kemeny constant was also tracked for the best solutions in each generation, however it was not included in the objective function. Its an interesting observation that the Kemeny constant tend to increase as we optimize vehicle density. Although it may seem counter intuitive at first, it's actually logical that as vehicle movement options is restricted in the network to minimize traffic congestion, the mean expected time to arrive at the different roads from an arbitrary road will increase as shorter path options get eliminated from the network. Figures 6.(a-d) show the network vehicle density and congestion in its starting stage and the final best solution from the GA run. Note that the optimized network achieves better vehicle density distribution among the entire network compared to the starting network.

5. Conclusion and Future Work

This article presented a novel approach to optimizing vehicle density distribution throughout the network while taking into consideration the complex interdependencies of such networks. The results show promising possibilities to utilize this approach as an automated traffic design system that can be deployed in crisis response management situations, were congestion reduction solutions are needed in short time frames, and the true optimality of the solution is of less importance since the impact time (the time the solution will be in effect) is typically limited to the presence of the event that triggered the situation. Using this approach coupled with a parallelized computing environment will allow for good-enough solutions to be generated considerably faster than what is possible through other methodologies such as agent based simulation and iterative design.

This line of research can be extended in several directions: 1) the exploration of other heuristic search methodologies such Simulated Annealing, Artificial Bee Colony, among others, 2) incorporating local search techniques to improve the resulting solution especially in early generation termination cases due to decision time-sensitivity, 3) multi-objective optimization to achieve best network balance solution where both vehicle density and the Kemeny constant are optimized, 4) investigating the environmental impact of such solutions if deployed for the long term, and 5) deriving a closed form mathematical model that is addressable by recent advances in convex optimization, specifically Semidefinite programing.
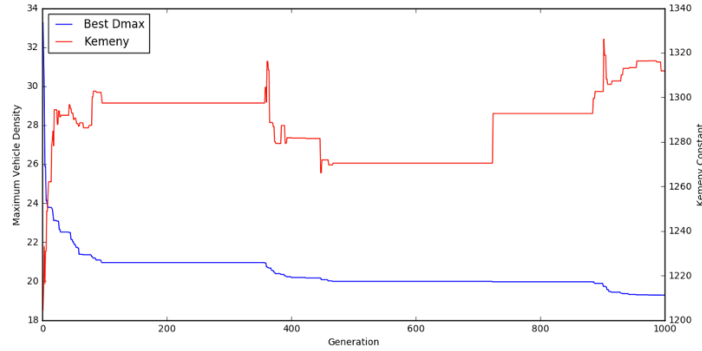
Figure 5. GA run results summary: Maximum Vehicle
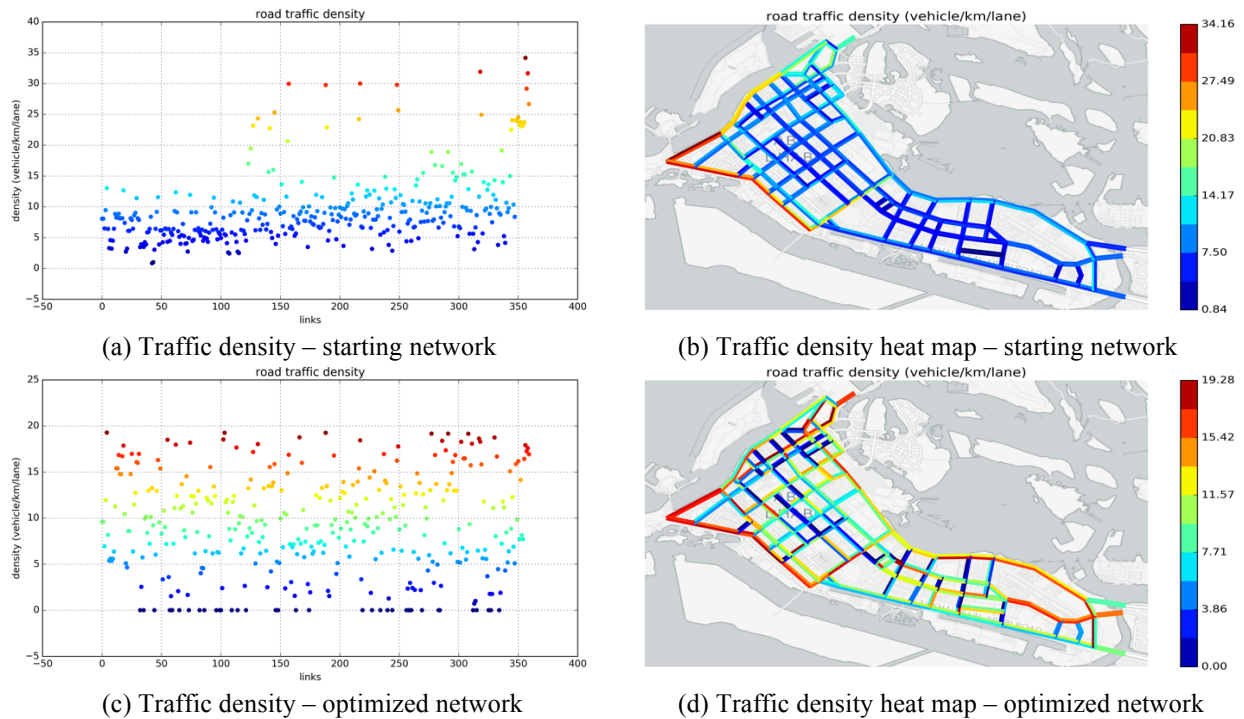Density and best fit network's Kemeny constant

(a) Traffic density – starting network

(b) Traffic density heat map – starting network

(c) Traffic density – optimized network

(d) Traffic density heat map – optimized network

Figure 6. Abu Dhabi traffic patter optimization results: starting and optimized networks

References

1. Dargay, J., Gately, D., & Sommer, M. (2007). Vehicle ownership and income growth, worldwide: 1960-2030. The Energy Journal, 143–170.
2. Abdulhai, B., & Kattan, L. (2004). Traffic engineering analysis. Handbook of Transportation Engineering, McGraw Hill.
3. Peeta, S., & Ziliaskopoulos, A. K. (2001). Foundations of dynamic traffic assignment: The past, the present and the future. *Networks and Spatial Economics*, *1*(3–4), 233–265.
4. Szeto, W., & Wong, S. (2012). Dynamic traffic assignment: model classifications and recent advances in travel choice principles. Open Engineering, 2(1), 1–18.
5. Jiang, B. (2009). Ranking spaces for predicting human movement in an urban environment. International Journal of Geographical Information Science, 23(7), 823–837.
6. Crisostomi, E., Kirkland, S., & Shorten, R. (2011). A Google-like model of road network dynamics and its application to regulation and control. International Journal of Control, 84(3), 633–651.

7. Silva A. and Wu N. (2012), Surveying Models in Urban Land Studies, Journal of Planning Literature 27(2) 139-152.
8. Zhang, H., & Gao, Z. (2007). Two-way road network design problem with variable lanes. Journal of Systems Science and Systems Engineering, 16(1), 50–61.
9. Long, J., Gao, Z., Zhang, H., & Szeto, W. Y. (2010). A turning restriction design problem in urban road networks. European Journal of Operational Research, 206(3), 569–578.
10. Long, J., Szeto, W. Y., & Huang, H.-J. (2014). A bi-objective turning restriction design problem in urban road networks. European Journal of Operational Research, 237(2),
11. Friesz, T. L. (1985). Transportation network equilibrium, design and aggregation: Key developments and research opportunities. *Transportation Research Part A: General*, *19*(5–6), 413–427.
12. Sasaki, T. (1965). Theory of traffic assignment through Absorbing Markov Process. In Proc. JSCE (Vol. 121, pp. 28–32).
13. Akamatsu, T. (1996). Cyclic flows, Markov process and stochastic traffic assignment. Transportation Research Part B: Methodological, 30(5), 369–386.
14. Bell, M. G. H., Schmoecker, J. D., Iida, Y., & Lam, W. H. K. (2002). Transit network reliability: an application of absorbing Markov chains. In Transportation and Traffic Theory in the 21st Century. Proceedings of the 15th International Symposium on Transportation and Traffic Theory.
15. Kurauchi, F., Bell, M. G. H., & Schmöcker, J.-D. (2003). Capacity Constrained Transit Assignment with Common Lines. Journal of Mathematical Modelling and Algorithms, 2(4), 309–327.
16. Faizrahnemoon, M., Schlote, A., Maggi, L., Crisostomi, E., & Shorten, R. (2015). A big-data model for multi-modal public transportation with application to macroscopic control and optimisation. International Journal of Control, 88(11), 2354–2368.
17. Sharma, S., & Mathew, T. V. (2011). Multiobjective Network Design for Emission and Travel-Time Trade-off for a Sustainable Large Urban Transportation Network. Environment and Planning B: Planning and Design, 38(3), 520–538.
18. Szeto, W. Y., Wang, Y., & Wong, S. C. (2014). The Chemical Reaction Optimization Approach to Solving the Environmentally Sustainable Network Design Problem. Computer-Aided Civil and Infrastructure Engineering, 29(2), 140–158.