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A Supply Chain Equilibrium Model with General Price-Dependent Demand

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ABSTRACT

The concept of supply chain equilibrium has been widely employed to solve real-life cases. Under this concept, decisions makers move simultaneously and compete in a noncooperative manner to achieve a supply chain network equilibrium. This paper proposes a supply chain network equilibrium model consisting of multiple raw material suppliers, manufacturers and retailers. Unlike previous studies, we assume that the demand for the product at each retail outlet is modeled as general stochastic functions of price that encompass additive-multiplicative demand models used in previous studies. Under general price-dependent demand functions, we derive the optimality conditions of suppliers, manufacturers and retailers, and establish that the governing equilibrium conditions can be formulated as a finite-dimensional variational inequality problem. The existence and uniqueness of the solution to the variational inequality are examined. A sensitivity analysis and a series of numerical tests are conducted to illustrate the analytical effects of demand distribution, model parameters, demand level and variability on quantity shipments, prices, and expected profits. Managerial insights are reported to show the impact of different types of demand functions and model parameters on the equilibrium solutions.

1. Introduction

The equilibrium concept in supply chains is drawn from network economics [1] and assumes a simultaneous move of the various decision-makers to achieve a supply chain network equilibrium. In the field of supply chain management, this concept is practically relevant and has been adopted to solve real-life cases. [2] develop a food supply chain equilibrium model for fresh product items, such as vegetables and fruits. The model was used to analyse various scenarios prior/during/-after a foodborn disease outbreak within the cantaloupe market in the United States. [3] propose a multitiered supply chain network equilibrium model for disaster relief with capacitated freight service provision. The model was applied to a case study on an international healthier crisis in western Africa to examine the impacts of adding a freight service provider and an humanitarian organization on the profits of freight service providers and on the costs incurred by the humanitarian organizations. Other relevant applications of the supply chain equilibrium concept includes electronic waste recycling [4], internet adverting [5], pharmaceutical products [6], green technology investment [7], and agricultural products [8].

Most of the studies dealing with supply chain equilibrium do not consider the effect of demand uncertainty on the equilibrium solutions. However, possessing relevant demand information such us density functions can assist operations supply chain managers to jointly compute optimal order quantities and prices before demand is realized. In this paper, we develop a new supply chain equilibrium model under demand uncertainty in which the demand for the product at each retailer is modeled by a general demand distribution and depends on all retailer prices and a on random variable independent of the price with increasing failure rate (IFR). This general demand model encompasses all common demand functions adopted in the literature [9,10] including additive linear, multiplicative isoelastic, power, logit, exponential, logarithmic, and mixed additive-multiplicative functions. The results of the supply chain equilibrium model are used to address the following research questions:

1. How do model parameters affect equilibrium solutions and expected profits of the supply chain members?
2. How the optimal quantity shipments, prices, and expected profits are influenced by the choice of the demand model?

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3. What is the effect of demand variability on each supply chain member's expected profit? Is this demand variability effect the same across different demand models?

Similar to [11], we adopt the concept of LSR elasticity and make realistic assumptions on the demand functions. Under these model assumptions, we demonstrate the pseudo-concavity of the retailers' expected profits as functions of prices. This allows us to characterize the equilibrium conditions of all raw material suppliers, manufacturers and retailers as a variational inequality in which they should determine their own optimal decision variables, given the optimal ones of competitors. As far as we are aware, this is the first supply chain equilibrium model to consider general price-dependent demand including all common demand functions adopted in the extant literature.

The main contribution of this paper is threefold. First, we develop a new supply chain equilibrium model that incorporates extended price-dependent stochastic demand functions and price competition among retailers. Second, we propose a new variational inequality formulation in which raw material suppliers, manufacturers and retailers should determine their optimal decision variables, given the optimal ones of competitors, and demonstrate the existence and uniqueness of the solution to this variational inequality. Third, through numerical tests and sensitivity analysis, we illustrate the analytic effects and practical managerial implications of different types of demand functions, model parameters, demand level, and demand variability on quantity shipments, prices, and expected profits.

The rest of this paper is organized as follows. Section 2 positions our research with respect to other contributions in the literature. Section 3 presents the optimality conditions of the raw material suppliers, manufacturers and retailers using the variational inequality theory. Section 4 provides the equilibrium conditions of the supply chain network model and its qualitative properties. Section 5 discusses examples of general demand models. Section 6 examines structural properties of the equilibrium solutions. Section 7 provides numerical examples to illustrate the effects of demand distribution, model parameters, demand level, and demand variability on quantity shipments, prices, and expected profits. Section 8 discusses some important managerial insights. Finally, Section 9 provides concluding remarks and ideas for future research. All proofs are provided in the appendices.

2. Positioning in literature

In determining multi-echelon supply chain optimal decisions, the game-theory-based framework used to model supply chain depends on the power relation between its members, see [12,13]. The first approach, assumes that decisions makers have similar powers, move simultaneously and compete in a noncooperative manner to achieve a supply chain network equilibrium. See [14] for review on this topic. Real-life applications of this approach in different fields were discussed in the Introduction section. Most studies have adopted a variational inequality (VI) formulation to characterize the equilibrium solutions of the various decision-makers. [15] were the first to develop an equilibrium model of a competitive supply chain network involving multiple manufacturers, retailers, and consumers in demand markets. This model provides the foundation of supply chain equilibrium models and has been extended by several authors to include capacity constraints [16,17], closed-loop supply chains [18,19], and multi-period supply chain networks [17,19,20].

The second approach, assumes that one member of the supply chain is more powerful and acts as a leader in the decision making process. A Stackelberg game theoretical framework with leaders and followers is used to find the optimal decisions. A thorough review of this approach can be found in [21,22]. Applications of this approach are found in various fields. [23] gave an application for a food supply chain dealing with pork meat industry. [24] uses this approach in a supply chain where it is desired to maximize profit and corporate social responsibility (CSR). In [25], the authors utilize social work donation and recycling investment as tools of corporate social responsibility and integrate the CSR investments into the optimization of a closed loop supply chain. [26] adopts this approach in a hospital supply chain.

Most literatures, dealing with Stackelberg Equilibrium in supply chains, notice that optimizing individual member objectives does not necessarily lead the optimality for the whole supply chain and that a centralized decision making leads to larger total supply chain profit. To solve such issue, many different type of contracts/incentives were introduced and aimed at inducing supply chain coordination. Literature dealing with supply chain coordination is quite extensive, see [27–29] for reviews. To cite some applications, one see that [30] uses price, rebate and returns supply contracts to coordinate supply chains while [31] adopts a hybrid all-unit quantity discount along a franchise fee contract for supply chain coordination. [32] examines the effect of channel leadership and information asymmetry on supply chain coordination. [33] uses product recycling and explores channel coordination in a socially responsible manufacturer-retailer closed-loop supply chain. The paper [34] proposes two hybrid contract bargaining processes that can be used for channel coordination of a supply chain that deals with a deteriorating product.

Our manuscript fits in the first approach and assumes that supply chain members move simultaneously in order to reach an equilibrium. Previous studies in this area utilize a projection-based algorithm to compute equilibrium shipments and prices but do not consider the effect of demand uncertainty on the equilibrium solutions. In fact, although the demand for a product may not be known with certainty but we may possess some information such as the density functions based on historical/forecasted data that allows decision makers to jointly determine order quantities and price before demand is realized. This is known in operations research literature as the newsvendor problem with pricing decisions [11,35,36].

The extant literature on the newsvendor problem with pricing (NVP) is extensive but mainly focuses on the additive and multiplicative models. In the additive and multiplicative demand cases, demand is represented as the sum and the product, respectively, of a deterministic price-dependent demand function and a random term that is independent of price. For the additive demand model, price affects the location of the demand distribution but not the demand variance while for the multiplicative case, price affects the scale of the distribution but not the coefficient of variation. [35] provide a comprehensive review of these two types of models when the mean demand is linear in the additive case and exponential (iso-elastic) in the multiplicative case. [36] and [37] study the NVP problem with additive and multiplicative demand models when the mean demand has increasing price elasticity (IPE) and the random noise has generalized increasing failure rate (GIFR). The authors prove that under these conditions, the expected profit of the newsvendor is unimodal or quasi-concave with respect to the price. In contrast to previous studies, [11] consider general price-dependent demand functions that include additive-multiplicative demand models as well as other relevant demand models such as logit, exponential, and power functions. Using a new measure, called the lost-sale rate (LSR) elasticity, they provide necessary and sufficient conditions for the NVP optimal policy under both coordinated and sequential decisions. Throughout the paper, the authors assume that riskless unconstrained revenue is strictly concave with respect to price. However, this assumption is not satisfied for logit, power, and iso-elastic demand functions which is a significant drawback of the paper. [38] consider a periodic review of the NPV problem under general price-dependent demands. For both the backorder and lost sales models, the objective is to maximize the expected profit over a finite selling horizon by coordinating the inventory and pricing decisions in each period. The authors utilize a new concept of upper-set and lower-set decreasing properties (USDP/LSDP) to derive sufficient conditions for the optimality of a base-stock list price policy based on the strict monotonicity of demand functions. Although the USDP/LSDP properties are considered to be small relaxations of the strict concavity of the riskless revenue, some demand functions such us the logit and exponential functions do not share these properties, limiting the applicability of the model.

As to supply chain equilibrium models under demand uncertainty, [39] develop a two-echelon model with multiple manufacturers and retailers. The retailers are faced with random demand and seek to maximize their expected profits with a penalty associated with a shortage and excess supply. The authors formulate the optimality conditions of the retailers as a variational inequality when the retailers first decide on the optimal amount to order from manufacturers. The equilibrium demand prices are then derived by assuming that the total quantity purchased by each retailer from all manufacturers is equal to the expected demand at that retail' outlet. This constitutes a main limitation because at equilibrium, the total actual demand is not necessarily equal to the total supply. The model of [39] was extended by [40] for multi-commodity flow and by [41] within closed-loop supply chains. [42] propose a dynamic supply chain equilibrium model for a closed-loop supply chain under uncertain and time-dependent demands and returns. The seasonality of demand is modeled by assuming that the expected value of the demand function is a cyclic function of time but independent of price. Using evolutionary variational inequality and projected dynamic systems, the authors derive dynamic equilibrium solutions. Results show that optimal production and transaction quantities are strongly affected by the seasonality of demand. [43] develop a decentralized closed-loop supply chain network model under random and price-sensitive demand and return. Using additive and multiplicative functions for both demand and return, the authors demonstrate the joint concavity of the retailers' and recovery centers' expected profits as functions of both shipment quantities and prices. This model has two main limitations. The first concerns the characteristics of the mean demand and return functions. The authors'assumptions limit the use of common demand and return functions, such as exponential and logit functions. The second issue is the competition among retailers and recovery centers. The authors assume that demand at each retail outlet and return at each recovery center depend only on the retailer's own price and the buy-back own price of the recovery center, respectively. In fact, in the presence of competition, the retailer's market demand is not only influenced by the retailer's own selling price, but also by the price set by competitors. Note that the NVP problem with retail competition was considered by [44] and [45] under additive linear demand functions; by [46] and [47] using multiplicative exponential demand functions; and by [48] using multiplicative demand models with increasing price elasticity (IPE). All of these models assume that demand at each retailer outlet depends on all retailer prices and the random noise has increasing failure rate/generalized increasing failure rate (IFR/GIFR).

3. The supply chain network model with general price-dependent demand

As shown in Figure 1, we consider a supply chain network consisting of N raw material suppliers who are involved in supplying one raw material to I manufacturers. The manufacturers produce a homogeneous product that can then be purchased by J retailers. We assume a one-to-one ratio between the raw material and product and this assumption can be easily relaxed by considering a non one-to-one ratio. Each retail outlet makes the product available to consumers in its own demand market. The links in the supply chain network denote transportation/transaction links. As assumed in the supply chain equilibrium literature, manufacturers must agree with the raw material suppliers as to the volume of shipments and retailers must agree with the manufacturers as to the purchasing prices which shall be determined using equilibrium conditions. In addition, all the supply chain members move simultaneously and compete in a noncooperative manner under the Cournot-Nash equilibrium framework, meaning decision makers will determine their own optimal decision variables, given the optimal ones of the competitors. The demand for the product at each retail outlet is modeled using a general demand distribution.

All indices, parameters and variables in the supply chain equilibrium network are defined as follows.

Indices

n: index of raw material suppliers in the SC network, $n = \{1,\ldots,N\}$.
i: index of manufacturers in the SC network, $i = \{1,\ldots,I\}$.
j: index of retailers in the SC network, $j = \{1,\ldots,J\}$.

Parameters

c_j: per-unit handling cost at retailer j.
λ_j^+: per-unit salvage value of having excess supply at retailer j.
λ_j^-: per-unit shortage cost of having excess demand at retailer j.

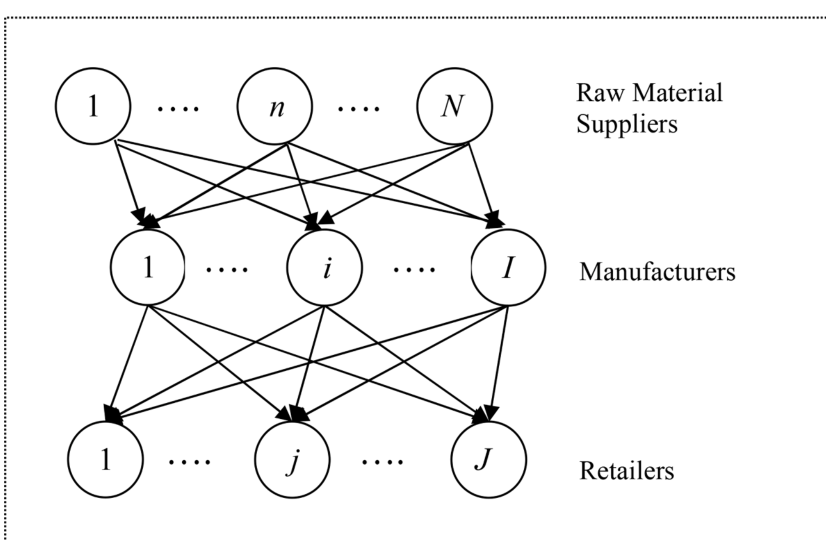


Fig. 1. The supply chain network

Variables

$\tilde{q}_{ni}$: nonnegative raw material shipment from supplier n to manufacturer i. Group the shipments of all the suppliers into the column vector $q_1 \in R_+^{NI}$.
q_{ij}: nonnegative product shipment from manufacturer i to retailer j. Group the shipments of all the manufacturers into the column vector $q_2 \in R_+^{IJ}$.
$\tilde{p}_{ni}$: selling price from supplier n to manufacturer i.
ρ_j: purchasing price at retailer outlet j. Group the prices of all the retailers into the column vector $\rho \in R_+^J$.
p_j: selling price at retailer outlet j. Group the prices of all the retailers into the column vector $p \in R_+^J$.

In the following subsections, we derive the optimality conditions of the raw material suppliers, manufacturers and retailers.

3.1. Raw Material Suppliers and their equilibrium conditions

Each raw material supplier n decides on the amount of raw material $\tilde{q}_{ni}$ to ship to each manufacturer i. Raw material supplier n incurs a procurement and a transaction cost, $c_{ni}(\tilde{q}_{ni})$, with each manufacturer i. Given the above cost, we can express the criterion of profit maximization for each raw material supplier n as:

$$\begin{aligned} \max_{\tilde{q}_{ni}} \Pi_n^S &= \sum_{i=1}^{I} \tilde{p}_{ni}^* \tilde{q}_{ni} - \sum_{i=1}^{I} c_{ni}\left(\tilde{q}_{ni}\right) \\ \text{subject to:} &\quad \tilde{q}_{ni} \geq 0. \end{aligned} \tag{1}$$

Equation (1) states that supplier n's profit equals sales revenue less costs associated with procurement and transaction. Note that $\tilde{p}_{ni}^*$ denote the optimal prices from each raw material supplier n to each manufacturer i. We will show later how to recover these optimal prices after solving the complete supply chain equilibrium model.

We assume that procurement and transaction cost functions for each raw material supplier are continuous and convex. Therefore, the optimality conditions for all raw material suppliers can be expressed simultaneously as the following variational inequality [43]: Determine $q_1^* \in R_+^{NI}$ satisfying:

$$\sum_{n=1}^{N} \sum_{i=1}^{I} \left[\frac{\partial c_{ni}\left(\tilde{q}_{ni}^*\right)}{\partial \tilde{q}_{ni}} - \tilde{p}_{ni}^* \right] \times \left[\tilde{q}_{ni} - \tilde{q}_{ni}^* \right] \geq 0 \qquad \forall q_1 \in R_+^{NI}. \tag{2}$$

3.2. Manufacturers and their equilibrium conditions

Each manufacturer i must decide on the amount of raw material $\tilde{q}_{ni}$ to get from each supplier n and the amount of product q_{ij} to ship to each retailer j. Raw material suppliers and manufacturers should agree on the quantities $\tilde{q}_{ni}$ and manufactures and retailers should also agree on the prices ρ_j which shall be determined using equilibrium conditions. The model assumes that retailer j pays the same price ρ_j to all manufacturers. This assumption is realistic since the model does not consider capacity constraints and any manufacturer i setting a higher price p_{ij} than the equilibrium price ρ_j^* would induce retailer j to not purchase any quantity from that manufacturer. Manufacturer i incurs a production and a transaction cost, $c_{ij}(q_{ij})$, with each retailer j. Given the above cost, we can write the objective of each manufacturer as:

$$\max_{\left(\tilde{q}_{ni}, q_{ij}\right)} \Pi_i^M = \sum_{j=1}^{J} \rho_j q_{ij} - \sum_{j=1}^{J} c_{ij}\left(q_{ij}\right) - \sum_{n=1}^{N} \tilde{p}_{ni}^* \tilde{q}_{ni} \tag{3}$$

$$\text{subject to:} \sum_{j=1}^{J} q_{ij} \leq \sum_{n=1}^{N} \tilde{q}_{ni}. \tag{4}$$

Equation (3) states that manufacturer i's profit equals sales revenue less costs associated with production and transaction, and payout to raw material suppliers. Constraint (4) states that the sum of all shipment quantities to retailers must be less or equal to the quantities procured from the raw material suppliers.

We assume that production and transaction cost functions for each manufacturer are continuous and convex. Therefore, the optimality conditions for all manufacturers can be expressed simultaneously as the following variational inequality: Determine $(q_1^*, q_2^*) \in \Lambda \subset R_+^{NI+IJ}$ satisfying:

$$\sum_{n=1}^{N} \sum_{j=1}^{J} \tilde{p}_{ni}^* \times \left[\tilde{q}_{ni} - \tilde{q}_{ni}^* \right] + \sum_{i=1}^{I} \sum_{j=1}^{J} \left[\frac{\partial c_{ij}\left(q_{ij}^*\right)}{\partial q_{ij}} - \rho_j \right] \times \left[q_{ij} - q_{ij}^* \right], \tag{5}$$

for all $(q_1, q_2) \in \Lambda$ where Λ is the convex set given by :

$$\Lambda = \left\{ (q_1, q_2) \in R_+^{NI+IJ} : \sum_{j=1}^{J} q_{ij} - \sum_{n=1}^{N} \tilde{q}_{ni} \leq 0, \ \forall 1 \leq i \leq I \right\}$$

3.3. Retailers and their equilibrium conditions

Each retailer j has to decide on the total amount to purchase from manufacturers and the selling price to consumers while simultaneously seeking to reach a Nash equilibrium under demand uncertainty. The demand for the product at each retailer j,$D_j(p, \epsilon_j)$, is assumed to follow a general demand distribution that depends on the whole vector price p and on a random variable ϵ_j, independent of p, defined on the range $[A_j, B_j]$. It can be seen that the classical additive demand model ($D_j(p, \epsilon_j) = d_j(p) + \epsilon_j$) and multiplicative model ($D_j(p, \epsilon_j) = d_j(p)\epsilon_j$) are simply special cases of the general demand model. If $s_j = \sum_{i=1}^{I} q_{ij}$ denotes the total supply at retailer j obtained from all the manufacturers, then if demand for the product does not exceed s_j, the revenue of retailer j is $p_j D_j(p, \epsilon_j)$ and each of the $s_j - D_j(p, \epsilon_j)$ leftovers is disposed at the unit salvage value λ_j^+. Alternatively, if demand exceeds s_j, then the revenue of retailer j is $p_j s_j$ and each of the $D_j(p, \epsilon_j) - s_j$ shortages incurs a per-unit shortage cost λ_j^-. Using the notation $Q_j = (q_{ij})_{i=1}^{I}$, the profit of retailer j, $W_j(Q_j, p_j)$, representing the difference between sales revenue and total costs, is given by

$$W_j\left(Q_j, p_j\right) = \begin{cases} p_j D_j\left(p, \epsilon_j\right) - c_j s_j - \rho_j s_j + \lambda_j^+ \left[s_j - D_j\left(p, \epsilon_j\right) \right] & \text{if } D_j\left(p, \epsilon_j\right) \leq s_j \\ p_j s_j - c_j s_j - \rho_j s_j - \lambda_j^- \left[D_j\left(p, \epsilon_j\right) - s_j \right] & \text{if } D_j\left(p, \epsilon_j\right) > s_j. \end{cases}$$

Assuming $D(p, x)$ to be strictly monotone in x and defining $z_j = z_j(p, s_j)$ as the unique solution of $D_j(p, z_j) = s_j$, the profit $W_j(Q_j, p_j)$ reduces to

$$\begin{aligned} W_j\left(s_j, p_j\right) &= \left(p_j + \lambda_j^- - c_j - \rho_j\right) s_j - \left(p_j + \lambda_j^- - \lambda_j^+\right) \left[s_j - D_j\left(p, \epsilon_j\right) \right] \mathbb{1}\left\{ \epsilon_j \leq z_j \right\} \\ &\quad - \lambda_j^- D_j\left(p, \epsilon_j\right). \end{aligned}$$

Each retailer j seeks to maximize the expected profit $\Pi_j(s_j, p_j) = E(W_j(s_j, p_j))$. More precisely, retailer j is trying to find s_j and p_j that maximize

$$\begin{aligned} \Pi_j\left(s_j, p_j\right) = &\left(p_j + \lambda_j^- - c_j - \rho_j\right) s_j - \left(p_j + \lambda_j^- - \lambda_j^+\right) s_j F_j\left(z_j\right) \\ &+ \left(p_j + \lambda_j^- - \lambda_j^+\right) \int_{A_j}^{z_j} D_j(p, x) f_j(x) dx - \lambda_j^- \int_{A_j}^{B_j} D_j(p, x) f_j(x) dx. \end{aligned} \tag{6}$$

To ensure the existence and uniqueness of an optimal solution, the following assumptions are needed

Assumption 1. For any retailer $j = 1, 2, \cdots, J$, the random variable ϵ_j

satisfies the following properties:

1. ϵ_j has a continuous distribution $F_j(x)$ with density $f_j(x)$.
2. The failure rate function of the ϵ_j's distribution, $r_j(x) = \frac{f_j(x)}{1-F_j(x)}$, is increasing.

As discussed in [43], classes of increasing failure rate (IFR) distributions include: Uniform, Normal (as well as truncated Normal at zero), Exponential, Gamma (with shape parameter $s \geq 1$), Beta (with parameters (r, s) both being ≥ 1), and Weibull distribution (with shape parameter $s \geq 1$).

Let $\eta_{jj}(p, x) = -\frac{p_j \frac{\partial D_j(p,x)}{\partial p_j}}{D_j(p,x)}$ denote the price elasticity of D_j. Price elasticity measures the percentage change in demand in response to a percentage change in retailer price p_j. We also define, for each $k \neq j$, the cross price elasticities $\eta_{jk}(p, x) = \frac{p_k \frac{\partial D_j(p,x)}{\partial p_k}}{D_j(p,x)}$ that measure the percentage change in demand in response to a percentage change in other retailer prices p_k. As adopted by [11], we define $\mathcal{E}_j(p, x) = -\frac{p_j \frac{\partial D_j(p,x)}{\partial p_j}}{\frac{\partial D_j(p,x)}{\partial x}} r_j(x)$ as the lost-sale rate (LSR) elasticity that measures the percentage change in the rate of lost sales with respect to the percentage change in price p_j for a given quantity x.

Assumption 2. For any retailer $j = 1, 2, \cdots, J$, demand $D_j(p, x)$ satisfies the following properties:

1. $\frac{\partial D_j(p,x)}{\partial x} \geq 0,\ \frac{\partial D_j(p,x)}{\partial p_j} \leq 0,\ \frac{\partial D_j(p,x)}{\partial p_k} \geq 0,\ \forall k \neq j,\ \frac{\partial D_j(p,x)}{\partial p_j} + \sum_{k \neq j} \frac{\partial D_j(p,x)}{\partial p_k} \leq 0.$
2. $\frac{\partial \eta_{jj}(p,x)}{\partial x} \leq 0,\ \frac{\partial \eta_{jk}(p,x)}{\partial x} \geq 0,\ \forall k \neq j,\ \frac{\frac{\partial \eta_{jj}(p,x)}{\partial x}}{p_j} + \sum_{k \neq j} \frac{\frac{\partial \eta_{jk}(p,x)}{\partial x}}{p_k} \leq 0.$
3. $\frac{\partial \mathcal{E}_j(p,x)}{\partial x} \geq 0,\ \frac{\partial \mathcal{E}_j(p,x)}{\partial p_j} \geq 0,\ \frac{\partial \mathcal{E}_j(p,x)}{\partial p_k} \leq 0,\ \forall k \neq j,\ \frac{\partial \mathcal{E}_j(p,x)}{\partial p_j} + \sum_{k \neq j} \frac{\partial \mathcal{E}_j(p,x)}{\partial p_k} \geq 0.$

Note that, Assumption 2.i) indicates that an increase in retailer j's price results in a decrease in the retailer's own demand while increasing that of competitors. This assumption also consider the substitution effect when demand does not increase under a uniform price increase [9]. The assumption also requires that demand at retailer j increases with the random quantity x. The same requirement is also found in [38].

Assumption 2.ii) stipulates that an increase in order quantity x will decrease price elasticity $\eta_{jj}(p, x)$ and increase price elasticity of other retailers $\eta_{jk}(p, x)$. The assumption encompasses competition requirements and considers the substitution and dominance effects among retailers.

Finally, Assumption 2.iii) requires that an increase in order quantity x will increase the LSR elasticity $\mathcal{E}_j(p,x)$. Also, an increase in retailer j's price will increase retailer j's LSR elasticity and decrease the LSR elasticity of other retailers. Moreover, the local price effect of a price change dominates the cross-price effect on local LSR elasticity. This assumption is consistent with those adopted by [11] and [38] but includes the dominance effect among retailers.

Assumption 3. For any retailer $j = 1, 2, \cdots, J$, demand $D_j(p, x)$ satisfies at least one of these assumptions:

1. $\frac{\partial^2 D_j(p,x)}{\partial p_j^2} \leq 0,\ \frac{\partial^2 D_j(p,x)}{\partial p_k p_j} \geq 0$ and $\frac{\partial^2 D_j(p,x)}{\partial p_j^2} + \sum_{k \neq j} \frac{\partial^2 D_j(p,x)}{\partial p_k p_j} \leq 0,\ \frac{\partial^2 D_j(p,x)}{\partial p_j^2}$ and $\frac{\partial^2 D_j(p,x)}{\partial p_j^2} + \sum_{k \neq j} \frac{\partial^2 D_j(p,x)}{\partial p_k p_j}$ are increasing in x, and $\frac{\partial^2 D_j(p,x)}{\partial p_k p_j}$ is decreasing in x.
2. $\frac{\partial^2 D_j(p,x)}{\partial p_j^2} \leq 0,\ \frac{\partial^2 D_j(p,x)}{\partial p_k p_j} \geq 0$ and $\frac{\partial^2 D_j(p,x)}{\partial p_j^2} + \sum_{k \neq j} \frac{\partial^2 D_j(p,x)}{\partial p_k p_j} \leq 0,\ \frac{\frac{\partial^2 D_j(p,x)}{\partial p_j^2}}{-\frac{\partial D_j(p,x)}{\partial p_j}}$ and $\frac{\frac{\partial^2 D_j(p,x)}{\partial p_j^2} + \sum_{k \neq j} \frac{\partial^2 D_j(p,x)}{\partial p_k p_j}}{-\frac{\partial D_j(p,x)}{\partial p_j}}$ are increasing in x, and $\frac{\frac{\partial^2 D_j(p,x)}{\partial p_k p_j}}{-\frac{\partial D_j(p,x)}{\partial p_j}}$ is decreasing in x.
3. $\frac{\partial^2 D_j(p,x)}{\partial p_j^2} \leq 0,\ \frac{\partial^2 D_j(p,x)}{\partial p_k p_j} \geq 0$ and $\frac{\partial^2 D_j(p,x)}{\partial p_j^2} + \sum_{k \neq j} \frac{\partial^2 D_j(p,x)}{\partial p_k p_j} \leq 0,\ \frac{\frac{\partial^2 D_j(p,x)}{\partial p_j^2}}{D_j(p,x)}$ and $\frac{\frac{\partial^2 D_j(p,x)}{\partial p_j^2} + \sum_{k \neq j} \frac{\partial^2 D_j(p,x)}{\partial p_k p_j}}{D_j(p,x)}$ are increasing in x, and $\frac{\frac{\partial^2 D_j(p,x)}{\partial p_k p_j}}{D_j(p,x)}$ is decreasing in x.
4. $\frac{\partial^2 D_j(p,x)}{\partial p_j^2} \geq 0,\ \frac{\partial^2 D_j(p,x)}{\partial p_k p_j} \leq 0$ and $\frac{\partial^2 D_j(p,x)}{\partial p_j^2} + \sum_{k \neq j} \frac{\partial^2 D_j(p,x)}{\partial p_k p_j} \geq 0,\ \frac{\partial^2 D_j(p,x)}{\partial p_j \partial x} \leq 0,\ \frac{\partial^2 D_j(p,x)}{\partial p_k \partial x} \geq 0$ and $\frac{\partial^2 D_j(p,x)}{\partial p_j \partial x} + \sum_{k \neq j} \frac{\partial^2 D_j(p,x)}{\partial p_k \partial x} \leq 0,\ \frac{\frac{\partial \mathcal{E}_j(p,x)}{\partial p_j}}{\mathcal{E}_j(p,x)}$ and $\frac{\frac{\partial \mathcal{E}_j(p,x)}{\partial p_j} + \sum_{k \neq j} \frac{\partial \mathcal{E}_j(p,x)}{\partial p_k}}{\mathcal{E}_j(p,x)}$ are decreasing in x, and $\frac{\frac{\partial \mathcal{E}_j(p,x)}{\partial p_k}}{\mathcal{E}_j(p,x)}$ is increasing in x, $\frac{\frac{\partial^2 D_j(p,x)}{\partial p_j \partial x}}{\frac{\partial D_j(p,x)}{\partial x}}$ and $\frac{\frac{\partial^2 D_j(p,x)}{\partial p_k \partial x}}{\frac{\partial D_j(p,x)}{\partial x}}$ are independent of x.

As seen in Section 5, all demand functions used in extant literature satisfy all conditions of Assumption 2. Moreover, each of these demand functions satisfy at least one of the above conditions of Assumption 3, allowing us to include all different types of demand functions in our supply chain equilibrium model.

To seek the optimal solution, note that taking the first derivative of function Π_j with respect to s_j and using the definition of z_j yields

$$\frac{\partial \Pi_j}{\partial s_j} = \left(p_j + \lambda_j^- - c_j - \rho_j\right) - \left(p_j + \lambda_j^- - \lambda_j^+\right) F_j\left(z_j\right).$$

It can be seen that, when $\lambda_j^+ \leq c_j + \rho_j \leq p_j$, the equation $\partial \Pi_j / \partial s_j = 0$ admits a solution $\bar{s}_j$ given by $\bar{s}_j = D_j(p, \bar{z}_j)$, where $\bar{z}_j = F_j^{-1}(x_j)$ with $x_j = (p_j + \lambda_j^- - c_j - \rho_j)/(p_j + \lambda_j^- - \lambda_j^+)$. Note that the condition $\lambda_j^+ \leq c_j + \rho_j$ amounts to the fact that the salvage value λ_j^+ should be less than or equal to the marginal cost $\rho_j + c_j$ and the condition $c_j + \rho_j \leq p_j$ ensures that retailer j is able to make nonnegative profit. Substituting $\bar{z}_j$ in (6) reduces the retailer problem to

$$\begin{aligned} \max_{p_j \in \Gamma_j(\rho_j)} \Pi_j(p_j) \quad = \quad & \left(p_j + \lambda_j^- - \lambda_j^+\right) \int_{A_j}^{\bar{z}_j} D_j(p, x) f_j(x) dx \\ & -\lambda_j^- \int_{A_j}^{B_j} D_j(p, x) f_j(x) dx, \end{aligned} \tag{7}$$

where $\Gamma_j(\rho_j) = \{p_j \in R_+ \,\big|\, \lambda_j^+ \leq c_j + \rho_j \leq p_j \leq \bar{p}_j\}$, and $\bar{p}_j$ is the maximum admissible price for retailer j. The next theorem shows that the retailer profit function $\Pi_j(p_j)$ is pseudo-concave.

Theorem 1. *If the conditions of Assumptions 1, 2, and 3 are satisfied, then the function $\Pi_j(p_j)$ is pseudo-concave in p_j.*

The proof is given in Appendix A.

Next, using Lemma 1 in [49] and Theorem 1 above, the optimality conditions for all retailers could be expressed simultaneously as the following variational inequality: Determine $p^* \in \Gamma(\rho) \subset R_+^J$ satisfying

$$\sum_{j=1}^{J} \left[-\left(p_j^* + \lambda_j^- - \lambda_j^+\right) \int_{A_j}^{\bar{z}_j} \frac{\partial D_j(p^*, x)}{\partial p_j} f_j(x) dx + \lambda_j^- \int_{A_j}^{B_j} \frac{\partial D_j(p^*, x)}{\partial p_j} f_j(x) dx - \int_{A_j}^{\bar{z}_j} D_j(p^*, x) f_j(x) dx - D_j\left(p^*, \bar{z}_j\right)\left(1 - F_j\left(\bar{z}_j\right)\right) \right] \times \left[p_j - p_j^*\right] \geq 0, \forall p \in \Gamma(\rho), \tag{8}$$

where $\bar{z}_j = F_j^{-1}(\bar{x}_j)$, $\bar{x}_j = \frac{p_j^* - c_j - \rho_j + \lambda_j^-}{p_j^* + \lambda_j^- - \lambda_j^+}$ and $\Gamma(\rho) = \bigotimes_{j=1}^{J} \Gamma_j(\rho_j)$.

4. Equilibrium conditions of the supply chain

4.1. Equilibrium conditions

As in the supply chain equilibrium literature, the sum of the optimality conditions for all raw material suppliers, as expressed by inequality (2), the sum of the optimality conditions for all manufacturers, as expressed by inequality (5) and the optimality conditions for all retailers, as expressed by inequality (8) must be satisfied. In addition, the amounts of the raw materials that the suppliers ship to the manufacturers must be equal to the shipments that the manufacturers accept from suppliers. Moreover, the amounts of the product that the manufacturers ship to the retailers must be equal to the total amounts purchased by the retailers, as expressed in the following condition:

$$s_j^* = D_j\left(p^*, F_j^{-1}\left(\frac{p_j^* - c_j - \rho_j^* + \lambda_j^-}{p_j^* + \lambda_j^- - \lambda_j^+}\right)\right) = \sum_{i=1}^{I} q_{ij}^* \tag{9}$$

Condition (9) states that when equilibrium price ρ_j^* that retailer j pays for the product is positive, then the supply s_j^* needed for at the retailer outlet is positive and must be equal to the total quantities purchased from all manufacturers. It can then be expressed as the following variational inequality: Determine $(q_2^*, p^*, \rho^*) \in R_+^{IJ} \times \Gamma \subset R_+^{IJ+2J}$:

$$\sum_{j=1}^{J}\left[\sum_{i=1}^{I} q_{ij}^* - D_j\left(p^*, F_j^{-1}\left(\frac{p_j^* - c_j - \rho_j^* + \lambda_j^-}{p_j^* + \lambda_j^- - \lambda_j^+}\right)\right)\right] \times \left[\rho_j - \rho_j^*\right] \geq 0 \tag{10}$$

$\forall(q_2, p, \rho) \in R_+^{IJ} \times \Gamma,\ \ \Gamma = \bigotimes_{j=1}^{J}\Gamma_j$ and $\Gamma_j = \{(p_j,\ \rho_j) \in R_+^2 \,\big|\, \lambda_j^+ \leq c_j + \rho_j \leq p_j \leq \bar{p}_j\}$.

The summation of inequalities (2), (5), (8), and (10) yields the following theorem:

Theorem 2. *The equilibrium conditions governing the supply chain model with general price-dependent demand are equivalent to the solution of the variational inequality problem given by: Determine* $(q_1^*, q_2^*, p^*, \rho^*) \in \Lambda \times \Gamma \subset R_+^{NI+IJ+2J}$ *satisfying*

$$\begin{aligned}
&\sum_{n=1}^{N}\sum_{i=1}^{I}\frac{\partial c_{ni}\left(\tilde{q}_{ni}^*\right)}{\partial \tilde{q}_{ni}} \times \left[\tilde{q}_{ni} - \tilde{q}_{ni}^*\right] + \sum_{i=1}^{I}\sum_{j=1}^{J}\left[\frac{\partial c_{ij}\left(q_{ij}^*\right)}{\partial q_{ij}} - \rho_j^*\right] \times \left[q_{ij} - q_{ij}^*\right] \\
&+ \sum_{j=1}^{J}\Bigg[-\left(p_j^* + \lambda_j^- - \lambda_j^+\right)\int_{A_j}^{z_j^*}\frac{\partial D_j(p^*, x)}{\partial p_j} f_j(x)dx + \lambda_j^-\int_{A_j}^{B_j}\frac{\partial D_j(p^*, x)}{\partial p_j} f_j(x)dx \\
&\quad - \int_{A_j}^{z_j^*} D_j(p^*, x) f_j(x)dx - D_j\left(p^*, z_j^*\right)\left(1 - F_j\left(z_j^*\right)\right)\Bigg] \times \left[p_j - p_j^*\right] \\
&+ \sum_{j=1}^{J}\left[\sum_{i=1}^{I} q_{ij}^* - D_j\left(p^*, z_j^*\right)\right] \times \left[\rho_j - \rho_j^*\right] \geq 0
\end{aligned} \tag{11}$$

$\forall(q_1, q_2, p, \rho) \in \Omega = \Lambda \times \Gamma$, *where* $z_j^* = F_j^{-1}\left(\frac{p_j^* - c_j - \rho_j^* + \lambda_j^-}{p_j^* + \lambda_j^- - \lambda_j^+}\right)$.

Proof. The proof follows from the standard variational inequality theory (e.g., [1]).□

4.2. Existence

Since the feasible set Ω is not compact, we need to impose an additional condition to guarantee the existence of a solution.

Let $\Omega_b \equiv \{(q_1, q_2, p, \rho,) | 0 \leq (q_1, q_2) \leq b,\ (p, \rho) \in \Gamma\}$, where $b = (b_1, b_2)$ and $q_1 \leq b_1$ and $q_2 \leq b_2$. Ω_b is a bounded closed convex subset of $\mathbb{R}_+^{NI+IJ+2J}$.

Theorem 3. *(Existence). Suppose that there exist positive constants R and S such that*

$$\frac{\partial c_{ni}\left(\tilde{q}_{ni}\right)}{\partial \tilde{q}_{ni}} \geq R,\ \forall q \text{ with } \tilde{q}_{ni} \geq S,\ \forall i, j. \tag{12}$$

$$\frac{\partial c_{ij}\left(q_{ij}\right)}{\partial q_{ij}} \geq R,\ \forall q \text{ with } q_{ij} \geq S,\ \forall i, j. \tag{13}$$

Then variational inequality (11) admits at least one solution.

Proof. The values of constants R and S are discussed in the existence proof in [50]. Following similar arguments on that proof, Assumptions (12) and (13) imply the existence of a constant b such that $(q_1, q_2) \leq b$ will guarantee the compactness of the set Ω_b and therefore the existence of a solution of variational inequality (11). Assumptions (12) and (13) can be economically justified as follows. When the raw material shipment $\tilde{q}_{ni}$ is large enough, one can expect the corresponding sum of the marginal costs associated with procurement and transaction to be large, which ensures (12). Similarly, when the product shipment q_{ij} is large, the corresponding sum of the marginal costs associated with production and transaction is expected to be large as well, which ensures (13).□

4.3. Uniqueness

Theorem 4. *(Uniqueness) Assume that cost functions,* c_{ni} *and* c_{ij}*, are strictly convex and that the conditions in Theorem 1 are satisfied for each* $1 \leq j \leq J$*, then variational inequality (11) admits a unique solution.*

The proof is provided in Appendix B.

5. Examples of demand functions

[10] gives a detailed list of demand functions adopted in the literature and presents a survey of empirical evidence showing the application of these demand functions in real industry sectors (sugar, yogurt, peanut butter, fashion, retail). The following are examples of classical demand functions, outlined in [10] and included in our framework.

- Additive Linear:
  $D_j(p, x) = x + a_j - b_j p_j + \sum_{k \neq j} c_{jk} p_k,\ \ a_j > 0,\ \ c_{jk} \geq 0,\ \ b_j > \sum_{k \neq j} c_{jk}$ and $A_j + a_j - b_j p_j + \sum_{k \neq j} c_{jk} p_k \geq 0$ to ensure nonnegative demand.
- Multiplicative Isoelastic (Power):
  $D_j(p, x) = a_j p_j^{-b_j} \prod_{k \neq j} p_k^{c_{jk}} x,\ a_j > 0,\ b_j > 1,\ c_{jk} \geq 0.$
- Logit:
  $D_j(p, x) = a_j \frac{e^{x - b_j p_j + \sum_{k \neq j} c_{jk} p_k} - 1}{1 + e^{x - b_j p_j + \sum_{k \neq j} c_{jk} p_k}},\ \ a_j > 0,\ \ c_{jk} \geq 0,\ \ b_j > \sum_{k \neq j} c_{jk},\ \ A_j - b_j p_j + \sum_{k \neq j} c_{jk} p_k > 0.$
- Exponential:
  $D_j(p, x) = e^{x + a_j - b_j p_j + \sum_{k \neq j} c_{jk} p_k},\ a_j > 0,\ c_{jk} \geq 0,\ b_j > \sum_{k \neq j} c_{jk}.$
- Logarithmic I:
  $D_j(p, x) = \ln\left(x + a_j - b_j p_j + \sum_{k \neq j} c_{jk} p_k\right),\ a_j > 0,\ c_{jk} \geq 0,\ b_j > \sum_{k \neq j} c_{jk},\ A_j + a_j - b_j p_j + \sum_{k \neq j} c_{jk} p_k > 1.$

- Logarithmic II:

  $D_j(p,x) = \ln\left(\left(a_j - b_j p_j + \sum_{k\neq j} c_{jk} p_k\right)^x\right)$, $a_j > 0$, $c_{jk} \geq 0$, $b_j > \sum_{k\neq j} c_{jk}$, $a_j - b_j p_j + \sum_{k\neq j} c_{jk} p_k > 1$.

- Mixed Additive-Multiplicative:

  $D_j(p,x) = \mu_j(p) + \sigma_j(p)x$, $\mu_j(p) + \sigma_j(p)A_j > 0$.

  $\frac{\partial \mu_j(p)}{\partial p_j} \leq 0$, $\frac{\partial \mu_j(p)}{\partial p_k} \geq 0$, $\forall k \neq j$, $\frac{\partial \mu_j(p)}{\partial p_j} + \sum_{k\neq j} \frac{\partial \mu_j(p)}{\partial p_k} \leq 0$.

  $\frac{\partial \sigma_j(p)}{\partial p_j} \leq 0$, $\frac{\partial \sigma_j(p)}{\partial p_k} \geq 0$, $\forall k \neq j$, $\frac{\partial \sigma_j(p)}{\partial p_j} + \sum_{k\neq j} \frac{\partial \sigma_j(p)}{\partial p_k} \leq 0$.

  $\frac{\partial \eta_{\sigma_j}}{\partial p_j} \geq 0$, $\frac{\partial \eta_{\sigma_j}}{\partial p_k} \leq 0$, $\forall k \neq j$, $\frac{\partial \eta_{\sigma_j}}{\partial p_j} + \sum_{k\neq j} \frac{\partial \eta_{\sigma_j}}{\partial p_k} \geq 0$.

  $\eta_{\mu_j}(p) \geq \eta_{\sigma_j}(p)$, $\quad \eta^k_{\mu_j}(p) \leq \eta^k_{\sigma_j}(p)$, $\forall k \neq j$, $\quad \frac{\eta_{\mu_j}(p)}{p_j} + \sum_{k\neq j} \frac{\eta^k_{\mu_j}(p)}{p_k} \geq \frac{\eta_{\sigma_j}(p)}{p_j} + \sum_{k\neq j} \frac{\eta^k_{\sigma_j}(p)}{p_k}$.

  $\frac{\frac{\partial^2 \mu_j}{\partial p_j^2}}{\frac{\partial \mu_j}{\partial p_j}} \geq \frac{\frac{\partial^2 \sigma_j}{\partial p_j^2}}{\frac{\partial \sigma_j}{\partial p_j}}$, $\frac{\frac{\partial^2 \mu_j}{\partial p_k \partial p_j}}{\frac{\partial \mu_j}{\partial p_j}} \leq \frac{\frac{\partial^2 \sigma_j}{\partial p_k \partial p_j}}{\frac{\partial \sigma_j}{\partial p_j}}$ $\forall k \neq j$, $\frac{\frac{\partial^2 \mu_j}{\partial p_j^2} + \sum_{k\neq j} \frac{\partial^2 \mu_j}{\partial p_k \partial p_j}}{\frac{\partial \mu_j}{\partial p_j}} \geq \frac{\frac{\partial^2 \sigma_j}{\partial p_j^2} + \sum_{k\neq j} \frac{\partial^2 \sigma_j}{\partial p_k \partial p_j}}{\frac{\partial \sigma_j}{\partial p_j}}$.

  Note that the last two assumptions are used in the literature. The first of these stipulates that the price elasticity of the mean demand $\mu_j(p)$ is greater than or equal to the price elasticity of the standard deviation demand $\sigma_j(p)$ [51,52]. This assumption encompasses competition requirements and considers the substitute and dominance effects among retailers. The second was examined by [53]. In our case, the assumption is generalized to competing retailers when substitution and dominance effects are required. It is important to note other related assumptions used in extant literature, namely the: log convexity of $\frac{\sigma_j(p)}{\mu_j(p)}$ considered by [54], and $\eta'_{\mu_j}(p) \geq \eta'_{\sigma_j}(p)$ whenever $\eta_{\mu_j}(p) \geq \eta_{\sigma_j}(p)$ considered by [51].

Each of the above demand functions satisfy Assumption 2 and at least one property of Assumption 3 and the proof of Theorem 1 (pseudo-convexity of the expected profits Π_j) holds under each of these properties allowing us to include all different types of demand functions in our supply chain equilibrium model. Note in particular that Assumption 3.i) is satisfied by the log function $D_j(p,x) = \ln\left(x + a_j - b_j p_j + \sum_{k\neq j} c_{jk} p_k\right)$. Assumption 3.ii) is valid for both log functions $D_j(p,x) = \ln\left(x + a_j - b_j p_j + \sum_{k\neq j} c_{jk} p_k\right)$ and $D_j(p, x) = \ln\left(\left(a_j - b_j p_j + \sum_{k\neq j} c_{jk} p_k\right)^x\right)$. Assumption 3.iii) is satisfied by the two logarithmic functions and the logit function $D_j(p,x) = a_j \frac{e^{x - b_j p_j + \sum_{k\neq j} c_{jk} p_k} - 1}{1 + e^{x - b_j p_j + \sum_{k\neq j} c_{jk} p_k}}$. Finally, Assumption 3.iv) is verified for the additive linear, multiplicative isoelastic, exponential, and mixed additive-multiplicative demand functions.

6. Sensitivity analysis

Using equations (1), (3), and (6), the total expected profit at equilibrium can be expressed as

$$\begin{aligned} \Pi &= \sum_{n=1}^{N} \Pi_n^S\left(\tilde{q}_{ni}^*\right) + \sum_{i=1}^{I} \Pi_i^M\left(q_{ij}^*\right) + \sum_{j=1}^{J} \Pi_j\left(q_{ij}^*, p_j^* \middle| p_{-j}^*\right) \\ &= \sum_{j=1}^{J}\left[p_j^* E\{D_j(p^*, \epsilon_j)\} - \Theta_j\left(p^*, z_j^*\right)\right] + \lambda_j^+ \Lambda_j\left(p^*, z_j^*\right) - \lambda_j^- \Theta_j\left(p^*, z_j^*\right) \\ &\quad - c_j \sum_{i=1}^{I} q_{ij}^*\Bigg] - \sum_{n=1}^{N}\sum_{i=1}^{I} c_{ni}\left(\tilde{q}_{ni}^*\right) - \sum_{i=1}^{I}\sum_{j=1}^{J} c_{ij}\left(q_{ij}^*\right) \end{aligned} \tag{14}$$

where $\Lambda_j(p, z_j) = \int_{A_j}^{z_j} (D_j(p, z_j) - D_j(p,x)) f_j(x) dx$ and

$$\Theta_j(p, z_j) = \int_{z_j}^{B_j} (D_j(p,x) - D_j(p, z_j)) f_j(x) dx$$

are the expected values of the leftover and shortage of retailer j, respectively, and where $D_j(p^*, z_j^*) = \sum_{i=1}^{I} q_{ij}^*$ and $z_j^* = F_j^{-1}\left(\frac{p_j^* - c_j - \rho_j^* + \lambda_j^-}{p_j^* + \lambda_j^- - \lambda_j^+}\right)$.

The following propositions illustrate the analytical effects of different types of model functions, model parameters (handling cost c_j, shortage cost λ_j^- and salvage value λ_j^+), demand level, and demand variability on the optimal solutions and the expected profits. All proofs are given in Appendix C.

Proposition 1 Impact of the handling cost c_j.

- *The optimal quantities $\tilde{q}_{ni}^*$ and q_{ij}^* decrease in c_j.*
- *The optimal prices p_j^* increase in c_j .*
- *The safety values z_j^* decrease in c_j .*
- *The raw material suppliers' profits Π_n^S, the manufacturers' profits Π_i^M, the retailers' profits Π_j, and the total profit Π decrease in c_j .*

Proposition 2 Impact of the unit shortage cost λ_j^-.

- *If $\frac{\partial D_j(p,x)}{\partial p_j}$ is increasing in x, the optimal quantities $\tilde{q}_{ni}^*$ and q_{ij}^* increase in λ_j^- .*
- *If $\frac{\partial D_j(p,x)}{\partial p_j}$ is decreasing in x, the prices p_j^* increase in λ_j^-.*
- *The safety values z_j^* increase in λ_j^-.*
- *The total profit Π decrease in λ_j^-.*

Proposition 3 Impact of the unit salvage value λ_j^+.

- *If $\frac{\partial D_j(p,x)}{\partial p_j}$ is decreasing in x, the optimal quantities $\tilde{q}_{ni}^*$ and q_{ij}^* increase in λ_j^+.*
- *If $\frac{\partial D_j(p,x)}{\partial p_j}$ is increasing in x, the prices p_j^* increase in λ_j^+.*
- *The safety values z_j^* increase in λ_j^+.*
- *The total profit Π increase in λ_j^+.*

Proposition 4 Impact of demand level. *Assume that for each retailer j, the demand level is controlled by a parameter a_j.*

- *The optimal quantities $\tilde{q}_{ni}^*$ and q_{ij}^* increase in a_j.*
- *The prices p_j^* increase in a_j.*

- *The raw material suppliers' profits Π_n^S, the manufacturers' profits Π_i^M, and the total profit Π increase in a_j.*

Proposition 5 Impact of demand variability. *Assume that for each retailer j, the demand variability is controlled by a parameter m_j.*

- *If the safety factors z_j^* are positive, the optimal quantities $\tilde{q}_{ni}^*$ and q_{ij}^* increase in m_j.*
- *If the safety factors z_j^* are negative, the optimal prices p_j^* decrease in m_j.*
- *The the total profit Π decreases in m_j.*

7. Numerical examples

A numerical study is carried out to show the effect of different model parameters on the equilibrium solution. As in [43], the extragradient algorithm of [55] is used to compute the solution of variational inequality (B.1). The algorithm is implemented in Matlab and has been successfully tested in previous studies [15,43].

After solving variational inequality (B.1), we can recover the equilibrium prices $\tilde{p}_{ni}^*$ using the optimality conditions of variational inequality problem (2). If there is a positive shipment quantity $\tilde{q}_{ni}^* > 0$, then $\tilde{p}_{ni}^* = \frac{\partial c_{ni}(\tilde{q}_{ni})}{\partial \tilde{q}_{ni}}$.

In our basic example, we consider a supply chain network with two raw material suppliers, two manufacturers and two retailers. The unit penalties of having excess supply/demand of retailers are set to $\lambda_j^+ = 2$, $\lambda_j^- = 2, \forall j = 1,2$. Also, the per-unit handling cost is set to $c_j = 40, \forall j = 1, 2$. The procurement and transaction cost functions faced by the suppliers and the production and transaction cost functions incurred by the manufacturers are given by:

$$c_{ni}\left(\tilde{q}_{ni}\right) = 1.5\left(\tilde{q}_{ni}\right)^2 + 10\left(\tilde{q}_{ni}\right) + 2,\ n = 1,\ i = 1,2.$$
$$c_{ni}\left(\tilde{q}_{ni}\right) = 1.5\left(\tilde{q}_{ni}\right)^2 + 9\left(\tilde{q}_{ni}\right) + 4,\ n = 2,\ i = 1,2.$$
$$c_{ij}\left(q_{ij}\right) = 1.5\left(q_{ij}\right)^2 + 9\left(q_{ij}\right) + 2,\ i = 1,\ j = 1,2.$$
$$c_{ij}\left(q_{ij}\right) = 1.5\left(q_{ij}\right)^2 + 11\left(q_{ij}\right) + 2,\ i = 2,\ j = 1,2.$$

Four models will be considered for the demand functions at retailer outlets:

- Additive Linear (Model 1):

$$D_j\left(p,\epsilon_j\right) = a_j - b_j p_j + \sum_{k\neq j} c_{jk} p_k + m\epsilon_j,$$

with $(a_1, a_2) = (290, 300)$ and for all $1 \le j \neq k \le 2$, $b_j = 2$ and $c_{jk} = 1$.
- Multiplicative Exponential (Model 2):

$$D_j\left(p,\epsilon_j\right) = \left(m_j + \epsilon_j\right) e^{a_j - b_j p_j + \sum_{k\neq j} c_{jk} p_k},$$

where $(m_1, m_2) = (290, 300)$ and for all $1 \le j \neq k \le 2$, $a_j = 1$, $b_j = 0.02$ and $c_{jk} = 0.01$.
- Mixed Linear-Exponential (Model 3):

$$D_j\left(p,\epsilon_j\right) = a_j - b_j p_j + \sum_{k\neq j} c_{jk} p_k + e^{\alpha_j - \beta_j p_j + \sum_{k\neq j} \gamma_{jk} p_k} \epsilon_j,$$

where $(a_1, a_2) = (290, 300)$ and for all $1 \le j \neq k \le 2$, $b_j = 2$, $c_{jk} = 1$, $\alpha_j = 5$, $\beta_j = 0.02$ and $\gamma_{jk} = 0.01$
- Logit(Model 4):

$$D_j\left(p,\epsilon_j\right) = m_j \left\{ e^{\epsilon_j + a_j - b_j p_j + \sum_{k\neq j} c_{jk} p_k} - 1 \right\} \Big/ \left\{ e^{\epsilon_j + a_j - b_j p_j + \sum_{k\neq j} c_{jk} p_k} + 1 \right\},$$

where $(m_1, m_2) = (100, 110)$ and for all $1 \le j \neq k \le 2$, $a_j = 5$, $b_j = 0.02$ and $c_{jk} = 0.01$.

For the above four models, ϵ_j for $j = 1, 2$, is chosen to have gamma distribution with shape parameter 2 and scale parameter 5. The distribution of ϵ_j is centered and reduced to have a mean of 0 and variance 1. This is carried out to avoid over-parameterization, since each of the above models contains parameters that can be used to control demand average and variability. Table 1 displays the optimal equilibrium solutions, the expected profits of all raw material suppliers, manufacturers and retailers, and the total supply chain profit. Comparing the quantities $\tilde{q}_{ni}^*$ and q_{ij}^*, we observe lower values in the mixed linear-exponential model and higher values in the multiplicative exponential model. This is mainly due to the impact of the rate of decrease of the expected demand with respect to retailer prices. The faster the demand decreases with respect to the retailer price p_j^*, the lower the quantities q_{ij}^* retailers will order from manufacturers. A decrease in q_{ij}^* will result in a decrease in the quantities $\tilde{q}_{ni}^*$, the expected profits Π_n^{S*}, Π_i^{M*} and Π_j^* and the total profit Π^*. Note that based on the current shortage and salvage parameters, we obtain negative values of z_j^* in all four demand models, resulting in a situation that favors shortages at each retail outlet.

The impact of demand distributions on the equilibrium solutions was also tested using the mixed additive-multiplicative demand model (Model 3) with the parameters specified above. Table 2 illustrates the results for the uniform distribution $\mathcal{U}(-\sqrt{3}, \sqrt{3})$, the reduced and truncated to $[-3, 3]$ normal distribution $\mathcal{N}(0, 1)$, and the centered and reduced gamma distribution with parameters (2,5) as above. Comparing

Table 1
Equilibrium solutions for different demand models

Model	Model 1	Model 2	Model 3	Model 4
q_{11}^*	25.75	26.82	23.79	28.17
q_{12}^*	26.47	27.09	24.68	29.83
q_{21}^*	25.42	26.49	23.45	27.84
q_{22}^*	26.14	26.76	24.35	29.50
$\tilde{q}_{11}^*$	25.95	26.79	24.07	28.84
$\tilde{q}_{12}^*$	25.61	26.46	23.73	28.50
$\tilde{q}_{21}^*$	26.28	27.12	24.40	29.17
$\tilde{q}_{22}^*$	25.95	26.79	24.07	28.84
p_1^*	239.67	269.88	231.29	298.22
p_2^*	242.53	270.68	234.47	299.32
ρ_1^*	174.10	179.84	162.56	190.03
ρ_2^*	176.25	180.64	165.24	195.00
$\tilde{p}_{11}^*$	87.84	90.37	82.20	96.51
$\tilde{p}_{12}^*$	86.84	89.37	81.20	95.51
$\tilde{p}_{21}^*$	87.84	90.37	82.20	96.51
$\tilde{p}_{22}^*$	86.84	89.37	81.20	95.51
z_1	-1.00	-0.85	-0.97	-0.76
z_2	-1.00	-0.85	-0.97	-0.79
Π_1^*	1296.34	2665.28	1250.67	3123.97
Π_2^*	1370.20	2692.02	1335.21	3103.80
Π_1^{M*}	2039.95	2174.08	1756.35	2519.49
Π_2^{M*}	1988.06	2120.50	1708.22	2461.82
Π_1^{S*}	1989.76	2122.53	1709.70	2461.85
Π_2^{S*}	2037.65	2172.11	1753.83	2515.52
Π	10721.96	13946.53	9513.98	16186.45

Table 2
Equilibrium solutions for different demand distributions

Distribution	$\mathcal{U}(-\sqrt{3},\sqrt{3})$	$\mathcal{N}(0,1)$	$GG(5,2,1)$
q_{11}^*	23.01	23.35	23.79
q_{12}^*	23.98	24.29	24.68
q_{21}^*	22.68	23.01	23.45
q_{22}^*	23.65	23.96	24.35
$\tilde{q}_{11}^*$	23.33	23.65	24.07
$\tilde{q}_{12}^*$	23.00	23.32	23.73
$\tilde{q}_{21}^*$	23.66	23.99	24.40
$\tilde{q}_{22}^*$	23.33	23.65	24.07
p_1^*	228.08	230.42	231.29
p_2^*	231.37	233.62	234.47
ρ_1^*	158.03	160.00	162.56
ρ_2^*	160.94	162.83	165.24
$\tilde{p}_{11}^*$	79.99	80.96	82.20
$\tilde{p}_{12}^*$	78.99	79.96	81.20
$\tilde{p}_{21}^*$	79.99	80.96	82.20
$\tilde{p}_{22}^*$	78.99	79.96	81.20
z_1	-1.25	-1.07	-0.97
z_2	-1.25	-1.07	-0.97
Π_1^*	1211.93	1132.32	1250.67
Π_2^*	1302.42	1230.49	1335.21
Π_1^{M*}	1651.07	1696.74	1756.35
Π_2^{M*}	1604.41	1649.43	1708.22
Π_1^{S*}	1605.78	1650.85	1709.70
Π_2^{S*}	1648.44	1694.15	1753.83
Π	9024.05	9053.99	9513.98

the results for the three demand distributions, we observe a slight increase in quantities $\tilde{q}_{ni}^*$ and q_{ij}^* from uniform to normal to generalized gamma distributions. This slight increase in $\tilde{q}_{ni}^*$ and q_{ij}^* results in small increases in the prices $\tilde{p}_{ni}^*$, ρ_j^* and p_j^*, the expected profits Π_n^{S*}, Π_i^{M*} and Π_j^*, and the total profit Π^*. These small changes in the optimal solutions are because all three demand distributions have a mean of 0 and a variance of 1, and differ only in the shape of their density functions. The impact of demand level and demand variability are discussed in sections 6.2 and 6.3.

7.1. Impact of model parameters

Here, we illustrate numerically the impact of model parameters (handling cost c_j, shortage cost λ_j^-, and salvage value λ_j^+) on the optimal solutions and the expected profits.

7.1.1. Impact of the handling cost c_j

We first investigate the effect of changing the handling cost c_j on the optimal quantities $\tilde{q}_{ni}^*$ and q_{ij}^*, optimal prices p_j^*, and expected profits. The logit model (Model 4) is used in the illustration and similar results are expected for other demand models. As proven in Proposition 1, an increase in c_j induces a decrease of the optimal quantities $\tilde{q}_{ni}^*$ and q_{ij}^*, and safety values z_j^* and an increase in the optimal prices p_j^* (see Table 3 for details). For the raw material suppliers, manufacturers, and retailers' profits, as shown in Proposition 1, their expected profits decrease with c_j which implies that the total profit decreases with c_j as displayed in Table 3.

Table 3
Impact of c_j on quantity shipments, prices, safety values and expected profits (Model 4)

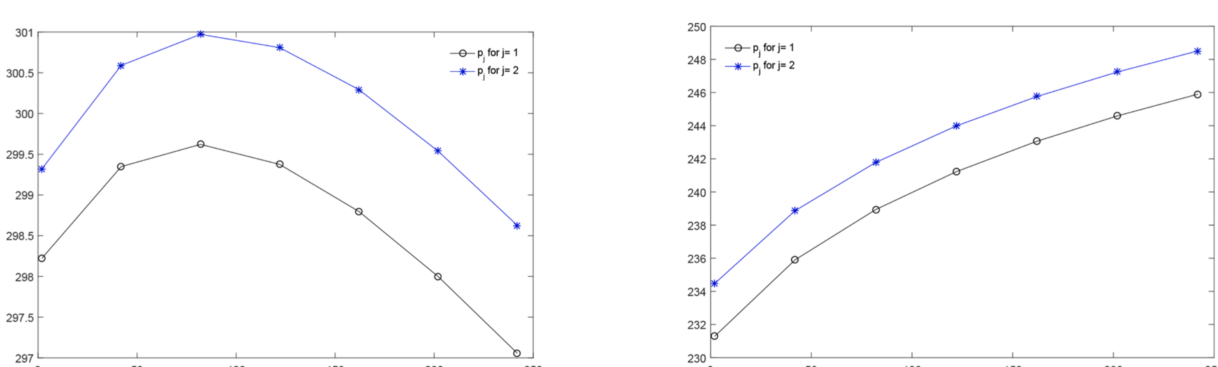
c_j	40	50	60	70	80	90	100
q_{11}^*	28.17	27.39	26.59	25.79	24.98	24.15	23.32
q_{12}^*	29.83	28.98	28.12	27.25	26.37	25.48	24.59
q_{21}^*	27.84	27.06	26.26	25.46	24.64	23.82	22.99
q_{22}^*	29.50	28.65	27.78	26.91	26.03	25.15	24.25
$\tilde{q}_{11}^*$	28.84	28.02	27.19	26.35	25.51	24.65	23.79
$\tilde{q}_{12}^*$	28.50	27.68	26.86	26.02	25.17	24.32	23.45
$\tilde{q}_{21}^*$	29.17	28.35	27.52	26.68	25.84	24.98	24.12
$\tilde{q}_{22}^*$	28.84	28.02	27.19	26.35	25.51	24.65	23.79
p_1^*	298.22	300.51	302.81	305.13	307.46	309.81	312.16
p_2^*	299.32	301.59	303.87	306.17	308.47	310.79	313.11
ρ_1^*	190.03	185.22	180.35	175.42	170.44	165.40	160.31
ρ_2^*	195.00	189.99	184.92	179.80	174.62	169.39	164.12
$\tilde{p}_{11}^*$	96.51	94.05	91.57	89.05	86.52	83.95	81.36
$\tilde{p}_{12}^*$	95.51	93.05	90.57	88.05	85.52	82.95	80.36
$\tilde{p}_{21}^*$	96.51	94.05	91.57	89.05	86.52	83.95	81.36
$\tilde{p}_{22}^*$	95.51	93.05	90.57	88.05	85.52	82.95	80.36
z_1	-0.76	-0.78	-0.81	-0.83	-0.85	-0.87	-0.89
z_2	-0.79	-0.81	-0.83	-0.85	-0.87	-0.89	-0.90
Π_1^*	3123.97	2897.52	2681.23	2474.65	2277.73	2090.16	1911.70
Π_2^*	3103.80	2876.54	2659.66	2452.85	2255.82	2068.31	1890.05
Π_1^{M*}	2519.49	2378.88	2240.74	2105.24	1972.57	1842.89	1716.39
Π_2^{M*}	2461.82	2322.85	2186.36	2052.54	1921.56	1793.59	1668.82
Π_1^{S*}	2461.85	2323.03	2186.70	2053.03	1922.18	1794.35	1669.70
Π_2^{S*}	2515.52	2375.07	2237.08	2101.73	1969.19	1839.65	1713.27
Π	16186.45	15173.90	14191.77	13240.03	12319.05	11428.95	10569.93

Table 4
Impact of λ_j^- on quantity shipments, prices, safety values and expected profits (Model 4)

λ_j^-	2	42	82	122	162	202	242
q_{11}^*	28.17	29.95	31.45	32.76	33.91	34.93	35.85
q_{12}^*	29.83	31.78	33.44	34.90	36.19	37.34	38.38
q_{21}^*	27.84	29.62	31.12	32.43	33.58	34.60	35.51
q_{22}^*	29.50	31.45	33.11	34.57	35.85	37.01	38.05
$\tilde{q}_{11}^*$	28.84	30.70	32.28	33.66	34.88	35.97	36.95
$\tilde{q}_{12}^*$	28.50	30.36	31.95	33.33	34.55	35.64	36.61
$\tilde{q}_{21}^*$	29.17	31.03	32.62	34.00	35.22	36.30	37.28
$\tilde{q}_{22}^*$	28.84	30.70	32.28	33.66	34.88	35.97	36.95
p_1^*	298.22	299.35	299.62	299.38	298.80	298.00	297.06
p_2^*	299.32	300.59	300.97	300.81	300.29	299.54	298.63
ρ_1^*	190.03	200.94	210.21	218.27	225.38	231.71	237.39
ρ_2^*	195.00	206.44	216.18	224.69	232.21	238.93	244.98
$\tilde{p}_{11}^*$	96.51	102.09	106.85	110.99	114.65	117.91	120.84
$\tilde{p}_{12}^*$	95.51	101.09	105.85	109.99	113.65	116.91	119.84
$\tilde{p}_{21}^*$	96.51	102.09	106.85	110.99	114.65	117.91	120.84
$\tilde{p}_{22}^*$	95.51	101.09	105.85	109.99	113.65	116.91	119.84
z_1	-0.76	-0.65	-0.55	-0.46	-0.39	-0.32	-0.26
z_2	-0.79	-0.67	-0.57	-0.49	-0.41	-0.35	-0.28
Π_1^*	3123.97	1916.34	844.09	-119.46	-992.24	-1787.54	-2515.28
Π_2^*	3103.80	1716.31	481.66	-630.59	-1640.68	-2563.22	-3409.41
Π_1^{M*}	2519.49	2854.41	3155.91	3430.83	3683.22	3915.95	4131.08
Π_2^{M*}	2461.82	2793.01	3091.34	3363.50	3613.45	3844.01	4057.18
Π_1^{S*}	2461.85	2792.58	3090.46	3362.16	3611.65	3841.75	4054.46
Π_2^{S*}	2515.52	2849.97	3151.02	3425.48	3677.42	3909.69	4124.36
Π	16186.45	14922.62	13814.48	12831.91	11952.81	11160.65	10442.40

7.1.2. *Impact of the shortage cost* λ_j^-

As proven in Proposition 2, the effect of the shortage cost λ_j^- on the optimal quantities $\tilde{q}_{ni}^*$ and q_{ij}^* and optimal prices p_j^* depends on the behavior of $\frac{\partial D_j(p,x)}{\partial p_j}$. Two demand models (Model 3 and Model 4) are used to illustrate the different patterns. In fact, it can be seen that when $\frac{\partial D_j(p,x)}{\partial p_j}$ is increasing in x (Model 4), the optimal quantities q_{ij}^* and $\tilde{q}_{ni}^*$ increase with λ_j^- as illustrated in Table 4. The sign of $\frac{dp_j^*}{d\lambda_j^-}$ depends on the model parameters. In our illustration, the sign of $\frac{dp_j^*}{d\lambda_j^-}$ depends on the value of λ_j^- as shown in Figure 2a (p_j^* increases for small λ_j^- and decreases for large λ_j^-). When $\frac{\partial D_j(p,x)}{\partial p_j}$ decreases in x (Model 3), we find that $\frac{dp_j^*}{d\lambda_j^-} \geq 0$ and that the signs of $\frac{d\tilde{q}_{ni}^*}{d\lambda_j^-}$ and $\frac{dq_{ij}^*}{d\lambda_j^-}$ depend on the model parameters. This behavior is illustrated in Table 5 and Figure 2b. Moreover, the safety value z_j^* increases with λ_j^- regardless of the demand function (Tables 4 and 5). Note that with large values of λ_j^-, positive values of z_j^* are obtained implying the likelihood of oversupply at each retail outlet to cope with high shortage costs. For the expected profits of raw material suppliers and manufacturers, as discussed in C.2, $\frac{\partial \Pi_n^S}{\partial \lambda_j^-}$ and $\frac{\partial \Pi_i^M}{\partial \lambda_j^-}$ have the same sign as $\frac{dq_{ij}^*}{d\lambda_j^-}$ (Tables 4, and 5). On the other hand, the sign of $\frac{\partial \Pi_j}{\partial \lambda_j^-}$ depends on the model parameters. In our illustration, for both Model 3 and 4 the expected profits of the retailers decrease with λ_j^- (Tables 4, and 5). For the total expected profit Π, Proposition 2 shows that the total profit Π decreases with λ_j^- as displayed in Tables 4, and 5.



(a) Selling prices p_j^* (Model 4) (b) Selling prices p_j^* (Model 3)

Fig. 2. Impact of λ_j^- on the prices p_j's

Table 5
Impact of λ_j^- on the quantity shipments, prices, safety values and expected profits (Model 3)

λ_j^-	2	42	82	122	162	202	242
q_{11}^*	23.79	23.73	23.66	23.60	23.55	23.50	23.47
q_{12}^*	24.68	24.67	24.63	24.59	24.55	24.52	24.49
q_{21}^*	23.45	23.39	23.32	23.26	23.21	23.17	23.13
q_{22}^*	24.35	24.34	24.29	24.25	24.22	24.19	24.16
$\tilde{q}_{11}^*$	24.07	24.03	23.98	23.93	23.88	23.84	23.81
$\tilde{q}_{12}^*$	23.73	23.70	23.64	23.59	23.55	23.51	23.48
$\tilde{q}_{21}^*$	24.40	24.36	24.31	24.26	24.22	24.18	24.15
$\tilde{q}_{22}^*$	24.07	24.03	23.98	23.93	23.88	23.84	23.81
p_1^*	231.29	235.91	238.94	241.22	243.06	244.59	245.90
p_2^*	234.47	238.87	241.78	243.98	245.76	247.24	248.51
ρ_1^*	162.56	162.27	161.89	161.56	161.28	161.04	160.83
ρ_2^*	165.24	165.10	164.81	164.54	164.30	164.09	163.91
$\tilde{p}_{11}^*$	82.20	82.09	81.93	81.78	81.65	81.53	81.44
$\tilde{p}_{12}^*$	81.20	81.09	80.93	80.78	80.65	80.53	80.44
$\tilde{p}_{21}^*$	82.20	82.09	81.93	81.78	81.65	81.53	81.44
$\tilde{p}_{22}^*$	81.20	81.09	80.93	80.78	80.65	80.53	80.44
z_1	-0.97	-0.69	-0.49	-0.34	-0.21	-0.11	-0.01
z_2	-0.97	-0.69	-0.50	-0.35	-0.22	-0.12	-0.02
Π_1^*	1250.67	854.83	542.59	281.56	56.57	-141.31	-317.96
Π_2^*	1335.21	986.16	707.24	472.29	268.73	88.98	-71.98
Π_1^{M*}	1756.35	1751.19	1743.20	1735.97	1729.73	1724.35	1719.67
Π_2^{M*}	1708.22	1703.13	1695.25	1688.12	1681.96	1676.66	1672.05
Π_1^{S*}	1709.70	1704.54	1696.62	1689.46	1683.29	1677.97	1673.34
Π_2^{S*}	1753.83	1748.60	1740.57	1733.31	1727.05	1721.66	1716.97
Π	9513.98	8748.45	8125.47	7600.70	7147.33	6748.30	6392.09

Table 6
Impact of λ_j^+ on the quantity shipments, prices, safety values and expected profits (Model 4)

λ_j^+	2	12	22	32	42	52	62
q_{11}^*	28.17	28.29	28.41	28.55	28.69	28.84	29.00
q_{12}^*	29.83	29.95	30.08	30.21	30.35	30.50	30.67
q_{21}^*	27.84	27.96	28.08	28.21	28.35	28.50	28.66
q_{22}^*	29.50	29.62	29.74	29.88	30.02	30.17	30.33
$\tilde{q}_{11}^*$	28.84	28.95	29.08	29.21	29.35	29.50	29.66
$\tilde{q}_{12}^*$	28.50	28.62	28.74	28.88	29.02	29.17	29.33
$\tilde{q}_{21}^*$	29.17	29.29	29.41	29.54	29.69	29.84	30.00
$\tilde{q}_{22}^*$	28.84	28.95	29.08	29.21	29.35	29.50	29.66
p_1^*	298.22	299.07	299.96	300.89	301.89	302.94	304.06
p_2^*	299.32	300.16	301.04	301.97	302.95	303.99	305.10
ρ_1^*	190.03	190.73	191.48	192.27	193.11	194.02	194.98
ρ_2^*	195.00	195.71	196.46	197.26	198.12	199.02	199.99
$\tilde{p}_{11}^*$	96.51	96.86	97.23	97.63	98.06	98.51	98.99
$\tilde{p}_{12}^*$	95.51	95.86	96.23	96.63	97.06	97.51	97.99
$\tilde{p}_{21}^*$	96.51	96.86	97.23	97.63	98.06	98.51	98.99
$\tilde{p}_{22}^*$	95.51	95.86	96.23	96.63	97.06	97.51	97.99
z_1	-0.76	-0.75	-0.73	-0.71	-0.70	-0.68	-0.66
z_2	-0.79	-0.77	-0.76	-0.74	-0.73	-0.71	-0.69
Π_1^*	3123.97	3130.03	3136.18	3142.39	3148.63	3154.88	3161.09
Π_2^*	3103.80	3108.95	3114.16	3119.36	3124.54	3129.65	3134.66
Π_1^{M*}	2519.49	2540.01	2561.88	2585.23	2610.22	2637.01	2665.81
Π_2^{M*}	2461.82	2482.11	2503.72	2526.81	2551.52	2578.01	2606.48
Π_1^{S*}	2461.85	2482.12	2503.74	2526.82	2551.51	2578.00	2606.47
Π_2^{S*}	2515.52	2536.03	2557.89	2581.24	2606.22	2633.01	2661.80
Π	16186.45	16279.26	16377.57	16481.85	16592.64	16710.56	16836.30

Table 7
Impact of λ_j^+ on the quantity shipments, prices, safety values and expected profits (Model 3)

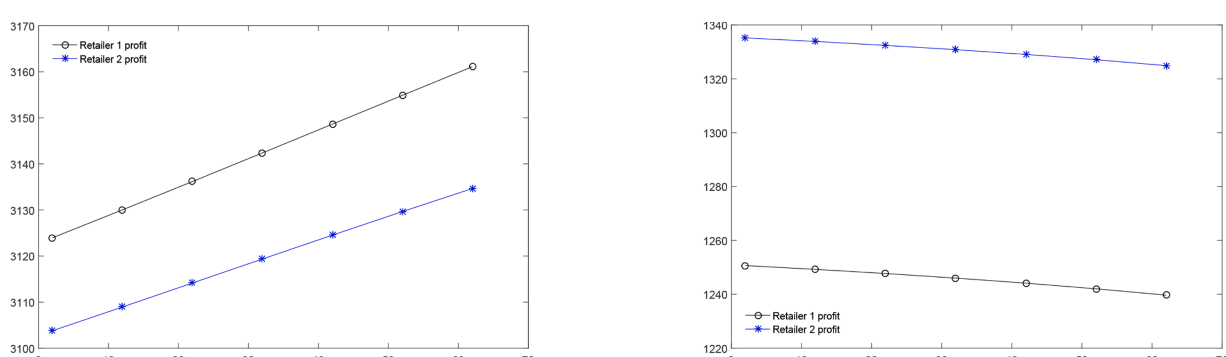
λ_j^+	2	12	22	32	42	52	62
q_{11}^*	23.79	23.82	23.85	23.88	23.92	23.96	24.00
q_{12}^*	24.68	24.71	24.73	24.77	24.80	24.83	24.87
q_{21}^*	23.45	23.48	23.51	23.55	23.58	23.62	23.67
q_{22}^*	24.35	24.37	24.40	24.43	24.46	24.50	24.54
$\tilde{q}_{11}^*$	24.07	24.09	24.12	24.16	24.19	24.23	24.27
$\tilde{q}_{12}^*$	23.73	23.76	23.79	23.82	23.86	23.90	23.94
$\tilde{q}_{21}^*$	24.40	24.43	24.46	24.49	24.52	24.56	24.60
$\tilde{q}_{22}^*$	24.07	24.09	24.12	24.16	24.19	24.23	24.27
p_1^*	231.29	231.43	231.58	231.74	231.92	232.11	232.32
p_2^*	234.47	234.61	234.76	234.91	235.08	235.27	235.47
ρ_1^*	162.56	162.73	162.91	163.11	163.33	163.56	163.82
ρ_2^*	165.24	165.40	165.58	165.76	165.97	166.19	166.43
$\tilde{p}_{11}^*$	82.20	82.28	82.37	82.47	82.57	82.69	82.81
$\tilde{p}_{12}^*$	81.20	81.28	81.37	81.47	81.57	81.69	81.81
$\tilde{p}_{21}^*$	82.20	82.28	82.37	82.47	82.57	82.69	82.81
$\tilde{p}_{22}^*$	81.20	81.28	81.37	81.47	81.57	81.69	81.81
z_1	-0.97	-0.95	-0.94	-0.93	-0.91	-0.90	-0.88
z_2	-0.97	-0.95	-0.94	-0.93	-0.91	-0.90	-0.88
Π_1^*	1250.67	1249.26	1247.72	1246.02	1244.14	1242.05	1239.70
Π_2^*	1335.21	1333.88	1332.44	1330.84	1329.08	1327.11	1324.92
Π_1^{M*}	1756.35	1760.37	1764.70	1769.39	1774.49	1780.06	1786.17
Π_2^{M*}	1708.22	1712.18	1716.45	1721.08	1726.11	1731.60	1737.62
Π_1^{S*}	1709.70	1713.67	1717.94	1722.57	1727.61	1733.10	1739.14
Π_2^{S*}	1753.83	1757.85	1762.19	1766.89	1771.99	1777.56	1783.68
Π	9513.98	9527.21	9541.44	9556.79	9573.41	9591.48	9611.23

7.1.3. Impact of the salvage value λ_j^+

The study of the effect of changing the salvage value λ_j^+ on the optimal quantities $\tilde{q}_{ni}^*$ and q_{ij}^* and optimal prices p_j^* is quite similar to that of the shortage value λ_j^-. The same models (Models 3 and 4) are used for illustration purposes. As shown in Proposition 3, when $\frac{\partial D_j(p,x)}{\partial p_j}$ increases in x, the optimal prices p_j^* increase with λ_j^+ and the signs of $\frac{d\tilde{q}_{ni}^*}{d\lambda_j^+}$ and $\frac{dq_{ij}^*}{d\lambda_j^+}$ depend on the model parameters as illustrated in Table 6. When $\frac{\partial D_j(p,x)}{\partial p_j}$ is decreasing in x, we obtain $\frac{d\tilde{q}_{ni}^*}{d\lambda_j^+} \geq 0,\ \frac{dq_{ij}^*}{d\lambda_j^+} \geq 0$ and the sign of $\frac{dp_j^*}{d\lambda_j}$ depends on the model parameters as displayed in Table 7. Moreover, the safety values z_j^* increase with λ_j^+ regardless of the demand function (Tables 6 and 7). For the expected profits, Proposition 3 has proved that the total profit increases with λ_j^+ as displayed in Tables 6 and 7. As discussed in C.3, $\frac{\partial \Pi_n^S}{\partial \lambda_j^+}$ and $\frac{\partial \Pi_i^M}{\partial \lambda_j^+}$ have the same sign as $\frac{dq_{ij}^*}{d\lambda_j^+}$ as illustrated in Tables 6 and 7. For the expected profits of the retailers, the sign of $\frac{\partial \Pi_j}{\partial \lambda_j^+}$ can be positive or negative depending on the model parameters. Figure 3a shows the case when the retailers' expected profits increase with λ_j^+ (Model 4) while Figure 3b shows the case when the retailers' expected profits decrease with λ_j^+ (Model 3).

7.2. Impact of demand level

The effect of demand level on the optimal quantity shipments, prices, and profits is explored. As mentioned in C.4, the mixed additive-



(a) Retailers' Profits (Model 4) (b) Retailers' Profits (Model 3)

Fig. 3. Impact of λ_j^+ on Retailers' Profit

Table 8
Impact of a_j on the quantity shipments, prices, safety values and expected profits

a_j	290	300	310	320	330	340	350
q_{11}^*	23.79	25.06	26.32	27.56	28.80	30.03	31.25
q_{12}^*	24.68	25.94	27.19	28.42	29.65	30.87	32.08
q_{21}^*	23.45	24.72	25.98	27.23	28.47	29.70	30.91
q_{22}^*	24.35	25.61	26.85	28.09	29.32	30.53	31.74
$\tilde{q}_{11}^*$	24.07	25.33	26.58	27.83	29.06	30.28	31.50
$\tilde{q}_{12}^*$	23.73	25.00	26.25	27.49	28.73	29.95	31.16
$\tilde{q}_{21}^*$	24.40	25.67	26.92	28.16	29.39	30.61	31.83
$\tilde{q}_{22}^*$	24.07	25.33	26.58	27.83	29.06	30.28	31.50
p_1^*	231.29	239.91	248.46	256.94	265.35	273.71	282.02
p_2^*	234.47	243.07	251.59	260.04	268.43	276.77	285.06
ρ_1^*	162.56	170.17	177.71	185.18	192.58	199.93	207.23
ρ_2^*	165.24	172.81	180.31	187.75	195.12	202.44	209.71
$\tilde{p}_{11}^*$	82.20	86.00	89.75	93.48	97.18	100.84	104.49
$\tilde{p}_{12}^*$	81.20	85.00	88.75	92.48	96.18	99.84	103.49
$\tilde{p}_{21}^*$	82.20	86.00	89.75	93.48	97.18	100.84	104.49
$\tilde{p}_{22}^*$	81.20	85.00	88.75	92.48	96.18	99.84	103.49
z_1	-0.97	-0.97	-0.97	-0.97	-0.97	-0.97	-0.97
z_2	-0.97	-0.97	-0.97	-0.97	-0.97	-0.97	-0.97
Π_1^*	1250.67	1380.81	1514.76	1652.60	1794.40	1940.23	2090.15
Π_2^*	1335.21	1467.47	1603.64	1743.79	1887.97	2036.25	2188.68
Π_1^{M*}	1756.35	1945.11	2141.51	2345.47	2556.90	2775.74	3001.95
Π_2^{M*}	1708.22	1894.45	2088.34	2289.81	2498.78	2715.18	2938.96
Π_1^{S*}	1709.70	1895.95	2089.86	2291.35	2500.33	2716.74	2940.53
Π_2^{S*}	1753.83	1942.61	2139.03	2343.00	2554.44	2773.30	2999.52
Π	9513.98	10526.41	11577.15	12666.01	13792.81	14957.44	16159.79

Table 9
Impact of m_j on the quantity shipments, prices, safety values and expected profits ($\lambda_j^- = 2$)

m_j	2	3	4	5	6	7	8
q_{11}^*	25.75	25.64	25.53	25.42	25.31	25.20	25.09
q_{12}^*	26.47	26.36	26.25	26.14	26.03	25.92	25.81
q_{21}^*	25.42	25.31	25.20	25.09	24.98	24.87	24.76
q_{22}^*	26.14	26.03	25.92	25.81	25.69	25.58	25.47
$\tilde{q}_{11}^*$	25.95	25.84	25.72	25.61	25.50	25.39	25.28
$\tilde{q}_{12}^*$	25.61	25.50	25.39	25.28	25.17	25.06	24.95
$\tilde{q}_{21}^*$	26.28	26.17	26.06	25.95	25.84	25.73	25.61
$\tilde{q}_{22}^*$	25.95	25.84	25.72	25.61	25.50	25.39	25.28
p_1^*	239.67	238.89	238.11	237.32	236.54	235.75	234.97
p_2^*	242.53	241.75	240.97	240.18	239.40	238.62	237.83
ρ_1^*	174.10	173.44	172.77	172.11	171.44	170.78	170.11
ρ_2^*	176.25	175.58	174.92	174.26	173.59	172.93	172.26
$\tilde{p}_{11}^*$	87.84	87.51	87.17	86.84	86.51	86.18	85.84
$\tilde{p}_{12}^*$	86.84	86.51	86.17	85.84	85.51	85.18	84.84
$\tilde{p}_{21}^*$	87.84	87.51	87.17	86.84	86.51	86.18	85.84
$\tilde{p}_{22}^*$	86.84	86.51	86.17	85.84	85.51	85.18	84.84
z_1	-1.00	-1.01	-1.01	-1.01	-1.01	-1.01	-1.01
z_2	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00
Π_1^*	1296.34	1278.51	1260.77	1243.12	1225.55	1208.08	1190.69
Π_2^*	1370.20	1351.93	1333.74	1315.64	1297.63	1279.70	1261.87
Π_1^{M*}	2039.95	2022.66	2005.44	1988.28	1971.19	1954.15	1937.19
Π_2^{M*}	1988.06	1970.99	1953.99	1937.05	1920.18	1903.37	1886.62
Π_1^{S*}	1989.76	1972.69	1955.69	1938.75	1921.88	1905.07	1888.32
Π_2^{S*}	2037.65	2020.36	2003.14	1985.98	1968.88	1951.85	1934.88
Π	10721.96	10617.15	10512.77	10408.82	10305.31	10202.23	10099.58

Table 10
Impact of m_j on the quantity shipments, prices, safety values and expected profits ($\lambda_j^- = 300$)

m_j	2	3	4	5	6	7	8
q_{11}^*	26.03	26.05	26.07	26.10	26.12	26.15	26.17
q_{12}^*	26.74	26.76	26.79	26.81	26.83	26.86	26.88
q_{21}^*	25.69	25.72	25.74	25.77	25.79	25.81	25.84
q_{22}^*	26.41	26.43	26.45	26.48	26.50	26.52	26.55
$\tilde{q}_{11}^*$	26.22	26.24	26.26	26.29	26.31	26.33	26.36
$\tilde{q}_{12}^*$	25.88	25.91	25.93	25.95	25.98	26.00	26.02
$\tilde{q}_{21}^*$	26.55	26.57	26.60	26.62	26.64	26.67	26.69
$\tilde{q}_{22}^*$	26.22	26.24	26.26	26.29	26.31	26.33	26.36
p_1^*	241.19	241.16	241.13	241.10	241.07	241.04	241.01
p_2^*	244.04	244.01	243.98	243.95	243.92	243.89	243.85
ρ_1^*	175.73	175.87	176.02	176.16	176.30	176.44	176.58
ρ_2^*	177.86	178.01	178.15	178.29	178.43	178.57	178.71
$\tilde{p}_{11}^*$	88.65	88.72	88.79	88.86	88.93	89.00	89.07
$\tilde{p}_{12}^*$	87.65	87.72	87.79	87.86	87.93	88.00	88.07
$\tilde{p}_{21}^*$	88.65	88.72	88.79	88.86	88.93	89.00	89.07
$\tilde{p}_{22}^*$	87.65	87.72	87.79	87.86	87.93	88.00	88.07
z_1	0.03	0.02	0.02	0.02	0.02	0.02	0.02
z_2	0.02	0.02	0.02	0.02	0.02	0.02	0.02
Π_1^*	902.88	688.26	473.68	259.15	44.67	-169.77	-384.15
Π_2^*	975.22	759.39	543.61	327.88	112.19	-103.44	-319.03
Π_1^{M*}	2082.49	2086.32	2090.13	2093.91	2097.65	2101.36	2105.05
Π_2^{M*}	2030.05	2033.84	2037.60	2041.33	2045.03	2048.69	2052.33
Π_1^{S*}	2031.76	2035.55	2039.31	2043.03	2046.73	2050.40	2054.04
Π_2^{S*}	2080.19	2084.03	2087.83	2091.61	2095.36	2099.07	2102.75
Π	10102.59	9687.39	9272.16	8856.91	8441.62	8026.32	7610.99

multiplicative model (Model 3) with $\mu_j(p) = a_j - b_j p_j + \sum_{k\neq j}^J c_{jk} p_k$ is used for illustration. The parameter a_j is used to control the demand level. From Proposition 4, the optimal quantities $\tilde{q}_{ni}^*$ and q_{ij}^* and the optimal price p_j^* increase with a_j as illustrated in Table 8. Note that the safety values z_j^* decrease with a_j (Table 8). Additionally, as proven in Proposition 4, the expected profits Π_n^{S*}, Π_i^{M*}, and Π^* increase with a_j as displayed in Table 8. Note that based on the model parameters, the expected profits of retailers also increase with a_j.

7.3. Impact of demand variability

The effect of demand variability on the optimal equilibrium is explored. The linear additive model (Model 1) with $D_j(p,x) = a_j - b_j p_j + \sum_{k\neq j}^J c_{jk} p_k + m_j x$ is used for this analysis and m_j is used to control demand variability. From Proposition 5, it can be seen that $\frac{d\tilde{q}_{ni}^*}{dm_j} \geq 0$ and $\frac{dq_{ij}^*}{dm_j} \geq 0$ when z_j^* is positive and their sign depend on the model parameters when z_j^* is negative. For the optimal prices, $\frac{dp_j^*}{dm_j} \leq 0$ when z_j^* is negative and its sign depends on the model parameters when z_j^* is positive. In our setting, the optimal quantities q_{ij}^* and the optimal prices p_j^* decrease with m_j when z_j^* is negative (Table 9). When z_j^* is positive, the optimal quantities $\tilde{q}_{ni}^*$ and q_{ij}^* increase and the optimal prices p_j^* decrease with m_j (Table 10). For the expected profits, Proposition 5 shows that the total profit Π decreases with m_j as displayed in Tables 9 and 10. Since the sign of $\frac{\partial \Pi_n^S}{\partial m_j}$ and $\frac{\partial \Pi_i^M}{\partial m_j}$ have the same sign as $\frac{dq_{ij}^*}{dm_j}$, the profits of the raw material suppliers and manufacturers decrease with m_j when z_j^* is negative (Table 9) and increase with m_j when z_j^* is positive (Table 10). Consequently, a positive value of z_j will induce retailers to order more from manufacturers whom profit from demand uncertainty when $z_j^* > 0$. Note that for both cases (negative and positive values of z_j^*), the expected profits of the retailers decrease with m_j (Tables 9 and 10).

8. Managerial insights

The summary of our key findings from the sensitivity analysis and numerical tests are as follows.

- The effect of model parameters on the equilibrium solutions depends on the type of demand model. For example, the effect of the shortage cost λ_j^- depends on the behavior of $\frac{\partial D_j(p,x)}{\partial p_j}$. For the logit model (Model 4), $\frac{\partial D_j(p,x)}{\partial p_j}$ is increasing in x and the optimal quantities q_{ij}^* and $\tilde{q}_{ni}^*$ increase with λ_j^- as illustrated in Table 4. The increase of q_{ij}^* and $\tilde{q}_{ni}^*$ will induce an increase of the expected profits of raw material suppliers and manufacturers (Table 4). On the other hand, using the mixed linear-exponential model (Model 3), $\frac{\partial D_j(p,x)}{\partial p_j}$ is decreasing in x and the optimal quantities q_{ij}^* and $\tilde{q}_{ni}^*$ decrease with λ_j^- resulting in a decrease of the expected profits of raw material suppliers and manufacturers (Table 5). Another example is the effect of the salvage value λ_j^+. With the logit model (Model 4), the retailers' expected profits increase with λ_j^+ as shown in Table 6. However, using the mixed linear-exponential model (Model 3), the retailers' expected profits decrease with λ_j^+ as illustrated in Table 7.
- For the same demand model, the effect of demand variability on the various supply chain members depends on the model parameters. Taking the linear additive model (Model 1) as an example, the total

expected profit is always decreasing with the demand variability. We expect the same behavior with the profits of retailers who are directly affected by an increase of demand variability (Table 9). However, the behavior of the expected profits of raw material suppliers and manufacturers depends on the model parameters. As illustrated in Table 9, the profits of the raw material suppliers and manufacturers decrease with the demand variability when the optimal safety values z_j^* are negative. When z_j^* are positive, the profits of the raw material suppliers and manufacturers increase with the demand variability (Table 10). Consequently, a positive value of z_j will induce retailers to order more from manufacturers and manufacturers to order more from raw material suppliers implying that both raw material suppliers and manufacturers would profit from demand uncertainty when $z_j^* > 0$.

Clearly, the results and insights obtained in our paper illustrate the importance of identifying the type of demand model in practice and generate interesting practical implications for managers and decision makers. First, the effect of a change in model parameters, like shortage cost and salvage value, on supply chain members is not straightforward and might result in different behaviors depending on the type of demand model. Therefore, all supply chain members should seek knowledge of the type of consumer demand model in their setting. Second, the effect of demand variability is only obvious in the case of the retailers who loose from an increased demand variability but might be counter intuitive for other supply chain members. In particular, depending on whether retailers best choice involves overstocking or not, manufacturers and raw material suppliers can either profit or loose from an increased demand variability. Finally, our new model can assist supply chain operations managers to quantify the effects of different types of demand functions, model parameters, demand level, and demand variability on quantity shipments, prices, and expected profits.

9. Conclusion

The concept of supply chain equilibrium has received increased attention in the supply chain management literature. Our study contributes to research in supply chain equilibrium by providing insights on how the type of demand function and model parameters affect the decisions and performance of the supply chain. In this paper, we develop a new supply chain equilibrium model in a network consisting of multiple suppliers, manufacturers and retailers who sell the product directly in their own demand markets. Demand uncertainty is modeled using a general demand model including additive, multiplicative, power, and logit functions. Moreover, to account for competitiveness, the demand for the product at each retail outlet is price-sensitive and depends on all retail prices.

Using a variational inequality approach, we derive the equilibrium conditions of raw material suppliers, manufacturers, and retailers. Existence and uniqueness of the equilibrium quantities and prices are discussed and an extragradient-based algorithm is proposed to solve the model. Sensitivity analysis and numerical examples illustrate the flexibility of the model and show the impact of demand function, model parameters, demand level and demand variability on the equilibrium shipments, prices, and expected profits.

Our model establishes the foundation for supply chain equilibrium problems under general price-dependent demand. The model is limited to a one-period setting with only two stages in the supply chain network. Additionally, our model does not consider capacity constraints and correlation of the retailers’uncertainties. Future research could extend the model to address these limitations. Other interesting avenues of future research include modeling the supply chain problem under general price-dependent demand within a Stackelberg equilibrium game framework and/or examining different type of contracts and incentives that could lead to supply chain coordination in networks with general demand functions.

Declaration of Competing Interest

The authors whose names are listed immediately below certify that they have NO affiliations with or involvement in any organization or entity with any financial interest (such as honoraria; educational grants; participation in speakers bureaus; membership, employment, consultancies, stock ownership, or other equity interest; and expert testimony or patent-licensing arrangements), or non-financial interest (such as personal or professional relationships, affiliations, knowledge or beliefs) in the subject matter or materials discussed in this manuscript.

Appendix A. Proof of Theorem 1

To prove Theorem 1, note that (8) yields

$$\frac{\partial \Pi_j}{\partial p_j} = \left(p_j + \lambda_j^- - \lambda_j^+\right) \int_{A_j}^{\bar{z}_j} \frac{\partial D_j(p,x)}{\partial p_j} f_j(x)dx - \lambda_j^- \int_{A_j}^{B_j} \frac{\partial D_j(p,x)}{\partial p_j} f_j(x)dx + \int_{A_j}^{\bar{z}_j} D_j(p,x) f_j(x)dx + D_j\left(p, \bar{z}_j\right)\left(1 - F_j\left(\bar{z}_j\right)\right), \tag{A.1}$$

where $\bar{z}_j = F_j^{-1}\left(\frac{p_j - c_j - \rho_j + \lambda_j^-}{p_j + \lambda_j^- - \lambda_j^-}\right)$. Using integration by parts and the definition of $\mathcal{E}_j(p,x)$, $\frac{\partial \Pi_j}{\partial p_j}$ can be rewritten as:

$$\frac{\partial \Pi_j}{\partial p_j} = \int_{A_j}^{\bar{z}_j} \left[1 - \frac{p_j - \lambda_j^+}{p_j} \mathscr{E}_j(p,x)\right] \frac{\partial D_j(p,x)}{\partial x} \left(1 - F_j(x)\right)dx + \int_{\bar{z}_j}^{B_j} \frac{\lambda_j^-}{p_j} \mathscr{E}_j(p,x) \frac{\partial D_j(p,x)}{\partial x} \left(1 - F_j(x)\right)dx + D_j\left(p, A_j\right) \tag{A.2}$$

$$= \int_{A_j}^{\bar{z}_j} \left[1 - \frac{p_j + \lambda_j^- - \lambda_j^+}{p_j} \mathscr{E}_j(p,x)\right] \frac{\partial D_j(p,x)}{\partial x} \left(1 - F_j(x)\right)dx + \int_{A_j}^{B_j} \frac{\lambda_j^-}{p_j} \mathscr{E}_j(p,x) \frac{\partial D_j(p,x)}{\partial x} \left(1 - F_j(x)\right)dx + D_j\left(p, A_j\right) \tag{A.3}$$

Using (A.1), (A.2) and (A.3), it can be seen that

$$\frac{\partial \Pi_j}{\partial p_j} = 0 \Longleftrightarrow \left(p_j + \lambda_j^- - \lambda_j^+\right) \int_{A_j}^{\bar{z}_j} \frac{\partial D_j(p,x)}{\partial p_j} f_j(x)dx - \lambda_j^- \int_{A_j}^{B_j} \frac{\partial D_j(p,x)}{\partial p_j} f_j(x)dx = -\int_{A_j}^{\bar{z}_j} D_j(p,x) f_j(x)dx - D_j\left(p, \bar{z}_j\right)\left(1 - F_j\left(\bar{z}_j\right)\right). \tag{A.4}$$

$$\frac{\partial \Pi_j}{\partial p_j} = 0 \Longleftrightarrow \int_{A_j}^{\bar{z}_j} \left[1 - \frac{p_j - \lambda_j^+}{p_j} \mathscr{E}_j(p,x)\right] \frac{\partial D_j(p,x)}{\partial x} \big(1 - F_j(x)\big) dx = -\int_{\bar{z}_j}^{B_j} \frac{\lambda_j^-}{p_j} \mathscr{E}_j(p,x) \frac{\partial D_j(p,x)}{\partial x} \big(1 - F_j(x)\big) dx - D_j\big(p, A_j\big). \tag{A.5}$$

$$\frac{\partial \Pi_j}{\partial p_j} = 0 \Longleftrightarrow \int_{A_j}^{\bar{z}_j} \left[1 - \frac{p_j + \lambda_j^- - \lambda_j^+}{p_j} \mathscr{E}_j(p,x)\right] \frac{\partial D_j(p,x)}{\partial x} \big(1 - F_j(x)\big) dx = -\int_{A_j}^{B_j} \frac{\lambda_j^-}{p_j} \mathscr{E}_j(p,x) \frac{\partial D_j(p,x)}{\partial x} \big(1 - F_j(x)\big) dx - D_j\big(p, A_j\big). \tag{A.6}$$

Since $\mathcal{E}_j(p,x)$ is increasing in x (Assumption 2.iii) and by (A.6),

$$\int_{A_j}^{\bar{z}_j} \left[1 - \frac{p_j + \lambda_j^- - \lambda_j^+}{p_j} \mathcal{E}_j(p,x)\right] \frac{\partial D_j(p,x)}{\partial x} \big(1 - F_j(x)\big) dx \le 0,$$

we get $\frac{p_j + \lambda_j^- - \lambda_j^+}{p_j} \mathcal{E}_j(p, \bar{z}_j) \ge 1$, which is equivalent to

$$\frac{\partial D_j\left(p, \bar{z}_j\right)}{\partial p_j} \left(1 - F_j\left(\bar{z}_j\right)\right) + \frac{\partial D_j\left(p, \bar{z}_j\right)}{\partial x} \frac{\left(1 - F_j\left(\bar{z}_j\right)\right)^2}{(p_j + \lambda_j^- - \lambda_j^+) f_j\left(\bar{z}_j\right)} \le 0. \tag{A.7}$$

From (A.1) and (A.2), the second derivative of Π_j with respect to p_j can be calculated in two ways:

$$\frac{\partial^2 \Pi_j}{\partial p_j^2}\Big|_{\frac{\partial \Pi_j}{\partial p_j} = 0} = \left(p_j + \lambda_j^- - \lambda_j^+\right) \int_{A_j}^{\bar{z}_j} \frac{\partial^2 D_j(p,x)}{\partial p_j^2} f_j(x) dx - \lambda_j^- \int_{A_j}^{B_j} \frac{\partial^2 D_j(p,x)}{\partial p_j^2} f_j(x) dx \tag{A.8}$$

$$+2 \int_{A_j}^{\bar{z}_j} \frac{\partial D_j(p,x)}{\partial p_j} f_j(x) dx + \frac{\partial D_j\left(p, \bar{z}_j\right)}{\partial p_j} \left(1 - F_j\left(\bar{z}_j\right)\right) \tag{A.9}$$

$$+\frac{\partial D_j\left(p, \bar{z}_j\right)}{\partial p_j} \left(1 - F_j\left(\bar{z}_j\right)\right) + \frac{\partial D_j\left(p, \bar{z}_j\right)}{\partial x} \frac{\left(1 - F_j\left(\bar{z}_j\right)\right)^2}{(p_j + \lambda_j^- - \lambda_j^+) f_j\left(\bar{z}_j\right)}, \tag{A.10}$$

or

$$\frac{\partial^2 \Pi_j}{\partial p_j^2}\Big|_{\frac{\partial \Pi_j}{\partial p_j} = 0} = \int_{A_j}^{\bar{z}_j} -\frac{p_j - \lambda_j^+}{p_j} \frac{\frac{\partial \mathscr{E}_j(p,x)}{\partial p_j}}{\mathscr{E}_j(p,x)} \mathscr{E}_j(p,x) \frac{\partial D_j(p,x)}{\partial x} \big(1 - F_j(x)\big) dx + \int_{\bar{z}_j}^{B_j} \frac{\lambda_j^-}{p_j} \frac{\frac{\partial \mathscr{E}_j(p,x)}{\partial p_j}}{\mathscr{E}_j(p,x)} \mathscr{E}_j(p,x) \frac{\partial D_j(p,x)}{\partial x} \big(1 - F_j(x)\big) dx \tag{A.11}$$

$$-\frac{\lambda_j^+}{p_j^2} \int_{A_j}^{\bar{z}_j} \mathscr{E}_j(p,x) \frac{\partial D_j(p,x)}{\partial x} \big(1 - F_j(x)\big) dx - \frac{\lambda_j^-}{p_j^2} \int_{\bar{z}_j}^{B_j} \mathscr{E}_j(p,x) \frac{\partial D_j(p,x)}{\partial x} \big(1 - F_j(x)\big) dx \tag{A.12}$$

$$+ \int_{A_j}^{\bar{z}_j} \frac{\frac{\partial^2 D_j(p,x)}{\partial p_j \partial x}}{\frac{\partial D_j(p,x)}{\partial x}} \left[1 - \frac{p_j - \lambda_j^+}{p_j} \mathcal{E}_j(p,x)\right] \frac{\partial D_j(p,x)}{\partial x} \big(1 - F_j(x)\big) dx + \int_{\bar{z}_j}^{B_j} \frac{\frac{\partial^2 D_j(p,x)}{\partial p_j \partial x}}{\frac{\partial D_j(p,x)}{\partial x}} \frac{\lambda_j^-}{p_j} \mathcal{E}_j(p,x) \frac{\partial D_j(p,x)}{\partial x} \big(1 - F_j(x)\big) dx + \frac{\partial D_j(p, A_j)}{\partial p_j} \tag{A.13}$$

$$+\frac{\partial D_j\left(p, \bar{z}_j\right)}{\partial p_j} \left(1 - F_j\left(\bar{z}_j\right)\right) + \frac{\partial D_j\left(p, \bar{z}_j\right)}{\partial x} \frac{\left(1 - F_j\left(\bar{z}_j\right)\right)^2}{(p_j + \lambda_j^- - \lambda_j^+) f_j\left(\bar{z}_j\right)}. \tag{A.14}$$

The rest of the proof is divided into two cases.

A1. First case: $\frac{\partial^2 D_j(p,x)}{\partial p_j^2} \le 0$

In this case, $\frac{\partial^2 \Pi_j}{\partial p_j^2}|_{\frac{\partial \Pi_j}{\partial p_j}=0} = A1 + A2 + A3$, where $A1$, $A2$, and $A3$ are given by the respective terms in (A.8), (A.9) and (A.10). Because $\frac{\partial D_j(p,x)}{\partial p_j} \le 0$ and $\frac{p_j+\lambda_j^- -\lambda_j^+}{p_j}\mathcal{E}_j(p,\bar{z}_j) \ge 1$, terms $A2$ and $A3$ are nonpositive. Note that when the shortage cost $\lambda_j^- = 0$, it is easy to show that $A1 \le 0$ since $\frac{\partial^2 D_j(p,x)}{\partial p_j^2} \le 0$. For $\lambda_j^- > 0$, the argument used to show that $A_1 \le 0$ depends on which part of Assumption 3 is satisfied. The details are outlined next in three subsections.

A1.1. $D_j(p,x)$ *satisfies Assumption 3.i*

In this case, $\frac{\partial^2 D_j(p,x)}{\partial p_j^2}$ is increasing in x, then

$$\begin{aligned} A1 &= \left(p_j - \lambda_j^+\right)\int_{A_j}^{\bar{z}_j}\frac{\partial^2 D_j(p,x)}{\partial p_j^2}f_j(x)dx - \lambda_j^-\int_{\bar{z}_j}^{B_j}\frac{\partial^2 D_j(p,x)}{\partial p_j^2}f_j(x)dx \\ &\le \frac{\partial^2 D_j\left(p,\bar{z}_j\right)}{\partial p_j^2}\left[\left(p_j+\lambda_j^- - \lambda_j^+\right)F_j\left(\bar{z}_j\right) - \lambda_j^-\right] = \frac{\partial^2 D_j\left(p,\bar{z}_j\right)}{\partial p_j^2}\left[p_j - c_j - \rho_j\right] \le 0. \end{aligned}$$

A1.2. $D_j(p,x)$ *satisfies Assumption 3.ii*

In this case, $\frac{\frac{\partial^2 D_j(p,x)}{\partial p_j^2}}{-\frac{\partial D_j(p,x)}{\partial p_j}}$ is increasing in x, then

$$A1 \le \frac{\frac{\partial^2 D_j\left(p,\bar{z}_j\right)}{\partial p_j^2}}{-\frac{\partial D_j\left(p,\bar{z}_j\right)}{\partial p_j}}\left[-\left(p_j-\lambda_j^+\right)\int_{A_j}^{\bar{z}_j}\frac{\partial D_j(p,x)}{\partial p_j}f_j(x)dx + \lambda_j^-\int_{\bar{z}_j}^{B_j}\frac{\partial D_j(p,x)}{\partial p_j}f_j(x)dx\right] = \frac{\frac{\partial^2 D_j\left(p,\bar{z}_j\right)}{\partial p_j^2}}{-\frac{\partial D_j\left(p,\bar{z}_j\right)}{\partial p_j}}\left[\int_{A_j}^{\bar{z}_j} D_j(p,x)f_j(x)dx + D_j\left(p,\bar{z}_j\right)\left(1-F_j\left(\bar{z}_j\right)\right)\right] \le 0.$$

The last equality holds because of equation (A.4).

A1.3. $D_j(p,x)$ *satisfies Assumption 3.iii*

In this case, $\frac{\frac{\partial^2 D_j(p,x)}{\partial p_j^2}}{D_j(p,x)}$ is increasing in x, then

$$A1 \le \frac{\frac{\partial^2 D_j\left(p,\bar{z}_j\right)}{\partial p_j^2}}{D_j\left(p,\bar{z}_j\right)}\left[\left(p_j-\lambda_j^+\right)\int_{A_j}^{\bar{z}_j} D_j(p,x)f_j(x)dx - \lambda_j^-\int_{\bar{z}_j}^{B_j} D_j(p,x)f_j(x)dx\right] = \frac{\frac{\partial^2 D_j\left(p,\bar{z}_j\right)}{\partial p_j^2}}{D_j\left(p,\bar{z}_j\right)}H\left(\lambda_j^-\right),$$

where $H(\lambda_j^-) = (p_j + \lambda_j^- - \lambda_j^+)\int_{A_j}^{\bar{z}_j} D_j(p,x)f_j(x)dx - \lambda_j^-\int_{A_j}^{B_j} D_j(p,x)f_j(x)dx$.

We have $H(0) = (p_j - \lambda_j^+)\int_{A_j}^{\bar{z}_j} D_j(p,x)f_j(x)dx > 0$ and

$$\begin{aligned} \frac{\partial H}{\partial \lambda_j^-} &= \left(p_j+\lambda_j^- - \lambda_j^+\right)D_j\left(p,\bar{z}_j\right)f_j\left(\bar{z}_j\right)\frac{\partial \bar{z}_j}{\partial \lambda_j^-} - \int_{\bar{z}_j}^{B_j} D_j(p,x)f_j(x)dx \\ &= \left(p_j+\lambda_j^- - \lambda_j^+\right)D_j\left(p,\bar{z}_j\right)f_j\left(\bar{z}_j\right)\frac{1-F_j\left(\bar{z}_j\right)}{\left(p_j+\lambda_j^- - \lambda_j^+\right)f_j\left(\bar{z}_j\right)} - \int_{\bar{z}_j}^{B_j} D_j(p,x)f_j(x)dx \\ &\le D_j\left(p,\bar{z}_j\right)\left(1-F_j\left(\bar{z}_j\right)\right) - D_j\left(p,\bar{z}_j\right)\left(1-F_j\left(\bar{z}_j\right)\right) = 0. \end{aligned}$$

Consequently, there exists $\lambda_j^0 > 0$ such that $H(\lambda_j^-) > 0$ for each $0 \le \lambda_j^- \le \lambda_j^0$, which implies that $A1 \le 0$ for all these values of λ^-.

A2. Second case: $\frac{\partial^2 D_j(p,x)}{\partial p_j^2} \ge 0$

According to Assumption 3.iv), $\frac{\partial^2 D_j(p,x)}{\partial p_j \partial x} \le 0$, $\frac{\frac{\partial \mathcal{E}_j(p,x)}{\partial p_j}}{\mathcal{E}_j(p,x)}$ is decreasing in x and $\frac{\frac{\partial^2 D_j(p,x)}{\partial p_j \partial x}}{\frac{\partial D_j(p,x)}{\partial x}}$ is independent of x. In this case, we have $\frac{\partial^2 \Pi_j}{\partial p_j^2}|_{\frac{\partial \Pi_j}{\partial p_j}=0} = B1 + B2 + B3 + B4$ where $B1$, $B2$, $B3$, and $B4$ are given by the respective terms in (A.11), (A.12), (A.13) and (A.14). Because $\frac{\partial D_j(p,x)}{\partial x} \geq 0$ and $\frac{p_j+\lambda_j^- - \lambda_j^+}{p_j}\mathcal{E}_j(p,\bar{z}_j) \geq 1$, terms $B2$ and $B4$ are nonpositive. Since $\frac{\frac{\partial \mathcal{E}_j(p,x)}{\partial p_j}}{\mathcal{E}_j(p,x)}$ is decreasing in x, we get:

$$
\begin{aligned}
B1 &= \int_{A_j}^{\bar{z}_j} -\frac{p_j-\lambda_j^+}{p_j}\frac{\frac{\partial \mathscr{E}_j(p,x)}{\partial p_j}}{\mathscr{E}_j(p,x)}\mathscr{E}_j(p,x)\frac{\partial D_j(p,x)}{\partial x}\left(1-F_j(x)\right)dx + \int_{\bar{z}_j}^{B_j}\frac{\lambda_j^-}{p_j}\frac{\frac{\partial \mathscr{E}_j(p,x)}{\partial p_j}}{\mathscr{E}_j(p,x)}\mathscr{E}_j(p,x)\frac{\partial D_j(p,x)}{\partial x}\left(1-F_j(x)\right)dx \\
&\leq \frac{\frac{\partial \mathscr{E}_j\left(p,\bar{z}_j\right)}{\partial p_j}}{\mathscr{E}_j\left(p,\bar{z}_j\right)}\left[\int_{A_j}^{\bar{z}_j} -\frac{p_j-\lambda_j^+}{p_j}\mathscr{E}_j(p,x)\frac{\partial D_j(p,x)}{\partial x}\left(1-F_j(x)\right)dx + \int_{\bar{z}_j}^{B_j}\frac{\lambda_j^-}{p_j}\mathscr{E}_j(p,x)\frac{\partial D_j(p,x)}{\partial x}\left(1-F_j(x)\right)dx\right] \\
&= \frac{\frac{\partial \mathscr{E}_j\left(p,\bar{z}_j\right)}{\partial p_j}}{\mathscr{E}_j\left(p,\bar{z}_j\right)}\left[-D_j\left(p,A_j\right) - \int_{A_j}^{\bar{z}_j}\frac{\partial D_j(p,x)}{\partial x}\left(1-F_j(x)\right)dx\right] \leq 0.
\end{aligned}
$$

The last equality holds because of equation (A.5).

Now, because $\frac{\frac{\partial^2 D_j(p,x)}{\partial p_j \partial x}}{\frac{\partial D_j(p,x)}{\partial x}}$ is independent of x, we obtain:

$$
B3 = \int_{A_j}^{\bar{z}_j}\frac{\frac{\partial^2 D_j(p,x)}{\partial p_j\partial x}}{\frac{\partial D_j(p,x)}{\partial x}}\left[1-\frac{p_j-\lambda_j^+}{p_j}\mathscr{E}_j(p,x)\right]\frac{\partial D_j(p,x)}{\partial x}\left(1-F_j(x)\right)dx + \int_{\bar{z}_j}^{B_j}\frac{\frac{\partial^2 D_j(p,x)}{\partial p_j\partial x}}{\frac{\partial D_j(p,x)}{\partial x}}\frac{\lambda_j^-}{p_j}\mathscr{E}_j(p,x)\frac{\partial D_j(p,x)}{\partial x}\left(1-F_j(x)\right)dx + \frac{\partial D_j\left(p,A_j\right)}{\partial p_j}
$$

$$
\begin{aligned}
&= \frac{-\frac{\partial^2 D_j\left(p,A_j\right)}{\partial p_j\partial x}}{\frac{\partial D_j\left(p,A_j\right)}{\partial x}}\left[\int_{A_j}^{\bar{z}_j}\left[\frac{p_j-\lambda_j^+}{p_j}\mathscr{E}_j(p,x)-1\right]\frac{\partial D_j(p,x)}{\partial x}\left(1-F_j(x)\right)dx - \int_{\bar{z}_j}^{B_j}\frac{\lambda_j^-}{p_j}\mathscr{E}_j(p,x)\frac{\partial D_j(p,x)}{\partial x}\left(1-F_j(x)\right)dx\right] + \frac{\partial D_j\left(p,A_j\right)}{\partial p_j} \\
&= \frac{-\frac{\partial^2 D_j\left(p,A_j\right)}{\partial p_j\partial x}}{\frac{\partial D_j\left(p,A_j\right)}{\partial x}}D_j\left(p,A_j\right) + \frac{\partial D_j\left(p,A_j\right)}{\partial p_j}.
\end{aligned}
$$

The last equality holds because of equation (A.5).

Since $\eta_{jj}(p,x)$ is decreasing in x (Assumption 2.ii), we have $\frac{-\frac{\partial^2 D_j(p,x)}{\partial p_j\partial x}}{\frac{\partial D_j(p,x)}{\partial x}} \leq \frac{-\frac{\partial D_j(p,x)}{\partial p_j}}{D_j(p,x)}$ which implies that

$$
B3 \leq \frac{-\frac{\partial D_j\left(p,A_j\right)}{\partial p_j}}{D_j\left(p,A_j\right)}D_j\left(p,A_j\right) + \frac{\partial D_j\left(p,A_j\right)}{\partial p_j} = 0.
$$

Consequently, $\frac{\partial^2 \Pi_j}{\partial p_j^2}\Big|_{\frac{\partial \Pi_j}{\partial p_j}=0} \leq 0$ in assumptions 3.i), 3.ii), 3.iii) and 3.iv) and therefore Π_j is pseudo-concave in p_j. □

Appendix B. Proof of Theorem 4

Variational inequality (11) can be rewritten in standard form as follows: determine $X^* \in \Omega$, such that

$$
\langle \mathcal{F}(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \Omega, \tag{B.1}
$$

where $X \equiv (q_1, q_2, p, \rho)$ and $\mathcal{F}(X) \equiv (\mathcal{F}_{ni}, \mathcal{F}_{ij}, \mathcal{F}_j^1, \mathcal{F}_j^2)$, with the specific components of $\mathcal{F}(X)$ being given by the respective functional terms preceding the multiplication signs in (11):

$$
\begin{aligned}
\mathcal{F}_{ni}(q_1,q_2,p,\rho) &= \frac{\partial c_{ni}\left(\tilde{q}_{ni}\right)}{\partial \tilde{q}_{ni}},\\
\mathcal{F}_{ij}(q_1,q_2,p,\rho) &= \frac{\partial c_{ij}\left(q_{ij}\right)}{\partial q_{ij}} - \rho_j,\\
\mathcal{F}^1_j(q_1,q_2,p,\rho) &= -\frac{\partial \Pi_j}{\partial p_j} = -\int_{A_j}^{z_j} D_j(p,x) f_j(x)dx - D_j\left(p,z_j\right)\left(1-F_j\left(z_j\right)\right),\\
&\quad -\left(p_j+\lambda_j^- - \lambda_j^+\right)\int_{A_j}^{z_j}\frac{\partial D_j(p,x)}{\partial p_j} f_j(x)dx + \lambda_j^-\int_{A_j}^{B_j}\frac{\partial D_j(p,x)}{\partial p_j} f_j(x)dx\\
\mathcal{F}^2_j(q_1,q_2,p,\rho) &= \sum_{i=1}^{I} q_{ij} - D_j\left(p,z_j\right), \quad \text{where } z_j = F_j^{-1}\left(\frac{p_j - c_j - \rho_j + \lambda_j^-}{p_j+\lambda_j^- - \lambda_j^-}\right)
\end{aligned} \tag{B.2}
$$

The equilibrium vector X^* is unique if $\mathcal{F}(X)=0|_{X=X^*}$ has a unique solution. The solution of $\mathcal{F}(X)=0|_{X=X^*}$ is closely related to the determinant of its Jacobian. Straightforward computations show that the Jacobian of $\mathcal{F}(X)$ is given by $M=\begin{pmatrix} A & B\\ C & D\end{pmatrix}$, where A is a $NI+IJ\times NI+IJ$ matrix, $B=-C^T$ is a $NI+IJ\times 2J$ matrix and D is a $2J\times 2J$ matrix defined as follows:

$$
A=\begin{pmatrix}
A^1_1 & O_{II} & \cdots & O_{II} & O_{JJ} & O_{JJ} & \cdots & O_{JJ}\\
O_{II} & A^1_2 & \cdots & O_{II} & O_{JJ} & O_{JJ} & \cdots & O_{JJ}\\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots\\
O_{II} & \cdots & O_{II} & A^1_N & O_{JJ} & O_{JJ} & \cdots & O_{JJ}\\
O_{II} & \cdots & O_{II} & O_{II} & A^2_1 & O_{JJ} & \cdots & O_{JJ}\\
O_{II} & \cdots & O_{II} & O_{II} & O_{JJ} & A^2_2 & \cdots & O_{JJ}\\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots\\
O_{II} & \cdots & O_{II} & O_{II} & O_{JJ} & O_{JJ} & \cdots & A^2_I
\end{pmatrix},
B=\begin{pmatrix}
O_{JJ} & O_{JJ}\\
O_{JJ} & O_{JJ}\\
\vdots & \vdots & \vdots\\
O_{JJ} & O_{JJ}\\
O_{JJ} & -\mathbb{I}_{JJ}\\
O_{JJ} & -\mathbb{I}_{JJ}\\
\vdots & \vdots & \vdots\\
O_{JJ} & -\mathbb{I}_{JJ}
\end{pmatrix},
D=\begin{pmatrix} D^1 & D^2\\ D^3 & D^4\end{pmatrix}
$$

where A^1_n is a $I\times I$ diagonal matrix with $(A^1_n)_{ii}=\frac{\partial^2 c_{ni}(\tilde{q}_{ni})}{\partial \tilde{q}^2_{ni}}$ $(1\le n\le N)$, A^2_i is a $J\times J$ diagonal matrix with $(A^2_i)_{jj}=\frac{\partial^2 c_{ij}(q_{ij})}{\partial q^2_{ij}}$ $(1\le i\le I)$, $\mathbb{I}_{JJ}$ is the identity matrix with rank J, O_{II} and O_{JJ} are $I\times I$ and $J\times J$ matrices of zeros, and matrices D^1, D^2, D^3 and D^4 are calculated as:

$$
D^1=\begin{pmatrix}
-\frac{\partial^2\Pi_1}{\partial p_1^2} & -\frac{\partial^2\Pi_1}{\partial p_2\partial p_1} & \cdots & -\frac{\partial^2\Pi_1}{\partial p_J\partial p_1}\\
-\frac{\partial^2\Pi_2}{\partial p_1\partial p_2} & -\frac{\partial^2\Pi_2}{\partial p_2^2} & \cdots & -\frac{\partial^2\Pi_2}{\partial p_J\partial p_2}\\
\vdots & \vdots & \ddots & \vdots\\
-\frac{\partial^2\Pi_J}{\partial p_1\partial p_J} & -\frac{\partial^2\Pi_J}{\partial p_2\partial p_J} & \cdots & -\frac{\partial^2\Pi_J}{\partial p_J^2}
\end{pmatrix},
D^2=\begin{pmatrix}
\gamma_1 & 0 & \cdots & 0\\
0 & \gamma_2 & \ddots & \vdots\\
\vdots & \ddots & \ddots & 0\\
0 & \cdots & 0 & \gamma_J
\end{pmatrix},
$$

$$
D^3=\begin{pmatrix}
-\gamma_1 & -\frac{\partial D_1}{\partial p_2} & \cdots & -\frac{\partial D_1}{\partial p_J}\\
-\frac{\partial D_2}{\partial p_1} & -\gamma_2 & \cdots & -\frac{\partial D_2}{\partial p_J}\\
\vdots & \vdots & \ddots & \vdots\\
-\frac{\partial D_J}{\partial p_1} & -\frac{\partial D_J}{\partial p_2} & \cdots & -\gamma_J
\end{pmatrix},
\text{ and } D^4=\begin{pmatrix}
\beta_1 & 0 & \cdots & 0\\
0 & \beta_2 & \ddots & \vdots\\
\vdots & \ddots & \ddots & 0\\
0 & \cdots & 0 & \beta_J
\end{pmatrix}
\text{ with } \beta_j=\frac{\frac{\partial D_j(p,z_j)}{\partial x}}{(p_j+\lambda_j^- - \lambda_j^+)f_j(z_j)} \text{ and } \gamma_j=\frac{\partial D_j(p,z_j)}{\partial p_j}+\frac{\frac{\partial D_j(p,z_j)}{\partial x}(1-F_j(z_j))}{(p_j+\lambda_j^- - \lambda_j^+)f_j(z_j)}.
$$

In order for the equilibrium vector to be unique, Theorem 1 of [49] requires $det(M)|_{X=X^*}>0$ for all equilibria X^*, along with two additional minor conditions: differentiability and boundary requirements. The former condition is met as assumed, and the boundary condition was needed to prove the existence of equilibrium. To proof the uniqueness of the equilibrium solution, it remains to be shown that $det(M)|_{X=X^*}>0$.

Since functions c_{ni} and c_{ij} are assumed to be strictly convex, all matrices A^1_n and A^2_i are invertible with positive determinants and therefore matrix A is also invertible with $det(A)>0$. Consequently, $det(M)=det(D-CA^{-1}B)det(A)$. Using the definitions of matrices B and C, it can be seen that $-CA^{-1}B=\begin{pmatrix} O_{JJ} & O_{JJ}\\ O_{JJ} & U\end{pmatrix}$, where U is a $J\times J$ diagonal matrix with $U_{jj}=\sum_{i=1}^{I}\frac{1}{\frac{\partial^2 c_{ij}(q_{ij})}{\partial q^2_{ij}}}$. Therefore, $D-CA^{-1}B=\begin{pmatrix} D^1 & D^2\\ D^3 & D^4+U\end{pmatrix}$. Since matrix D^4+U is a diagonal matrix with positive elements equal to $\delta_j=\beta_j+U_{jj}$, then $det(D-CA^{-1}B)=det(D^1-D^2(D^4+U)^{-1}D^3)det(D^4+U)$. Using the definitions of matrices D^1, D^2, D^3, and D^4+U, it can be seen that the elements of the matrix $N=D^1-D^2(D^4+U)^{-1}D^3$ are given by $N_{jj}=-\frac{\partial^2\Pi_j}{\partial p_j^2}+\frac{\gamma_j^2}{\delta_j}$, $\forall 1\le j\le J$ and $N_{jk}=-\frac{\partial^2\Pi_j}{\partial p_k\partial p_j}+\frac{\gamma_j}{\delta_j}\frac{\partial D_j}{\partial p_k}$, $\forall k\ne j$. The proof is complete if we can establish that $det(N)|_{X=X^*}>0$. Using Theorem 4 in [56] it can be seen that all principal minors of N are positive if N is diagonally dominant with positive diagonal entries and negative off diagonal entries. The rest of the proof is devoted to establishing that N satisfies these properties when $X=X^*$.

It follows from Theorem 1, that $\frac{\partial^2\Pi_j}{\partial p_j^2}|_{\frac{\partial\Pi_j}{\partial p_j}=0} < 0$, therefore $N_{jj}|_{X=X^*} = -\frac{\partial^2\Pi_j}{\partial p_j^2} + \frac{\gamma_j^2}{\delta_j} > 0$. Using (A.1) and (A.2), the second derivative of Π_j with respect to p_k and p_j can be written as

$$\frac{\partial^2\Pi_j}{\partial p_k p_j}|_{\frac{\partial\Pi_j}{\partial p_j}=0} = \int_{A_j}^{\bar{z}_j} \frac{\partial D_j(p,x)}{\partial p_k} f_j(x)dx + \frac{\partial D_j\left(p,\bar{z}_j\right)}{\partial p_k}\left(1 - F_j\left(\bar{z}_j\right)\right) \tag{B.3}$$

$$+\left(p_j + \lambda_j^- - \lambda_j^+\right)\int_{A_j}^{\bar{z}_j} \frac{\partial^2 D_j(p,x)}{\partial p_k p_j} f_j(x)dx - \lambda_j^- \int_{A_j}^{B_j} \frac{\partial^2 D_j(p,x)}{\partial p_k p_j} f_j(x)dx \tag{B.4}$$

or

$$\frac{\partial^2\Pi_j}{\partial p_k p_j}|_{\frac{\partial\Pi_j}{\partial p_j}=0} = \int_{A_j}^{\bar{z}_j} -\frac{p_j - \lambda_j^+}{p_j}\frac{\frac{\partial \mathscr{E}_j(p,x)}{\partial p_k}}{\mathscr{E}_j(p,x)}\mathscr{E}_j(p,x)\frac{\partial D_j(p,x)}{\partial x}\left(1 - F_j(x)\right)dx + \int_{\bar{z}_j}^{B_j} \frac{\lambda_j^-}{p_j}\frac{\frac{\partial \mathscr{E}_j(p,x)}{\partial p_k}}{\mathscr{E}_j(p,x)}\mathscr{E}_j(p,x)\frac{\partial D_j(p,x)}{\partial x}\left(1 - F_j(x)\right)dx \tag{B.5}$$

$$+\int_{A_j}^{\bar{z}_j} \frac{\frac{\partial^2 D_j(p,x)}{\partial p_k \partial x}}{\frac{\partial D_j(p,x)}{\partial x}}\left[1 - \frac{p_j - \lambda_j^+}{p_j}\mathscr{E}_j(p,x)\right]\frac{\partial D_j(p,x)}{\partial x}\left(1 - F_j(x)\right)dx + \int_{\bar{z}_j}^{B_j} \frac{\frac{\partial^2 D_j(p,x)}{\partial p_k \partial x}}{\frac{\partial D_j(p,x)}{\partial x}}\frac{\lambda_j^-}{p_j}\mathscr{E}_j(p,x)\frac{\partial D_j(p,x)}{\partial x}\left(1 - F_j(x)\right)dx + \frac{\partial D_j(p,A_j)}{\partial p_k}. \tag{B.6}$$

To study the sign of N_{jk}, we distinguish two cases.

First case: $\frac{\partial^2 D_j(p,x)}{\partial p_k p_j} \geq 0$

In this case, $\frac{\partial^2\Pi_j}{\partial p_k p_j}|_{\frac{\partial\Pi_j}{\partial p_j}=0} = C1 + C2$, where $C1$ and $C2$ are given by the respective terms in (B.4) and (B.3). Because $\frac{\partial D_j(p,x)}{\partial p_k} \geq 0$, term $C2$ is nonnegative. If $D_j(p,x)$ satisfies Assumption 3.i), $\frac{\partial^2 D_j(p,x)}{\partial p_k p_j}$ is decreasing in x and

$$\begin{aligned} C1 &= \left(p_j + \lambda_j^- - \lambda_j^+\right)\int_{A_j}^{\bar{z}_j} \frac{\partial^2 D_j(p,x)}{\partial p_k p_j} f_j(x)dx - \lambda_j^- \int_{A_j}^{B_j} \frac{\partial^2 D_j(p,x)}{\partial p_k p_j} f_j(x)dx \\ &\geq \frac{\partial^2 D_j\left(p,\bar{z}_j\right)}{\partial p_k p_j}\left[\left(p_j + \lambda_j^- - \lambda_j^+\right)F_j\left(\bar{z}_j\right) - \lambda_j^-\right] = \frac{\partial^2 D_j\left(p,\bar{z}_j\right)}{\partial p_k p_j}\left[p_j - c_j - \rho_j\right] \geq 0. \end{aligned}$$

If $D_j(p,x)$ satisfies Assumption 3.ii) or 3.iii), the same arguments used in the proof of Theorem 1 will imply that term $C1$ is non-negative.

Second case: $\frac{\partial^2 D_j(p,x)}{\partial p_k p_j} \leq 0$

According to Assumption 3.iv), $\frac{\partial^2 D_j(p,x)}{\partial p_k \partial x} \geq 0$, $\frac{\frac{\partial \mathcal{E}_j(p,x)}{\partial p_k}}{\mathcal{E}_j(p,x)}$ is increasing in x and $\frac{\frac{\partial^2 D_j(p,x)}{\partial p_k \partial x}}{\frac{\partial D_j(p,x)}{\partial x}}$ is independent of x. In this case, we have $\frac{\partial^2\Pi_j}{\partial p_k \partial p_j}|_{\frac{\partial\Pi_j}{\partial p_j}=0} = D1 + D2$, where $D1$ and $D2$ are given by the respective terms in (B.5) and (B.6). Since $\frac{\frac{\partial \mathcal{E}_j(p,x)}{\partial p_k}}{\mathcal{E}_j(p,x)}$ is increasing in x, we get

$$\begin{aligned} D1 &\geq \frac{\frac{\partial \mathscr{E}_j\left(p,\bar{z}_j\right)}{\partial p_k}}{\mathscr{E}_j\left(p,\bar{z}_j\right)}\left[\int_{A_j}^{\bar{z}_j} -\frac{p_j - \lambda_j^+}{p_j}\mathscr{E}_j(p,x)\frac{\partial D_j(p,x)}{\partial x}\left(1 - F_j(x)\right)dx + \int_{\bar{z}_j}^{B_j}\frac{\lambda_j^-}{p_j}\mathscr{E}_j(p,x)\frac{\partial D_j(p,x)}{\partial x}\left(1 - F_j(x)\right)dx\right] \\ &= \frac{\frac{\partial \mathscr{E}_j\left(p,\bar{z}_j\right)}{\partial p_k}}{\mathscr{E}_j\left(p,\bar{z}_j\right)}\left[-D_j(p,A_j) - \int_{A_j}^{\bar{z}_j}\frac{\partial D_j(p,x)}{\partial x}\left(1 - F_j(x)\right)dx\right] \geq 0. \end{aligned}$$

The last equality holds because $\frac{\partial \mathcal{E}_j(p,\bar{z}_j)}{\partial p_k} \leq 0$ (Assumption 2.iii) and equation (A.5)).

Now, because $\frac{\frac{\partial^2 D_j(p,x)}{\partial p_k \partial x}}{\frac{\partial D_j(p,x)}{\partial x}}$ is independent of x, we obtain

$$\begin{aligned} D2 &= \frac{-\frac{\partial^2 D_j\left(p,A_j\right)}{\partial p_k \partial x}}{\frac{\partial D_j\left(p,A_j\right)}{\partial x}}\left[\int_{A_j}^{\bar{z}_j}\left[\frac{p_j - \lambda_j^+}{p_j}\mathscr{E}_j(p,x) - 1\right]\frac{\partial D_j(p,x)}{\partial x}\left(1 - F_j(x)\right)dx - \int_{\bar{z}_j}^{B_j}\frac{\lambda_j^-}{p_j}\mathscr{E}_j(p,x)\frac{\partial D_j(p,x)}{\partial x}\left(1 - F_j(x)\right)dx\right] + \frac{\partial D_j\left(p,A_j\right)}{\partial p_j} \\ &= \frac{-\frac{\partial^2 D_j\left(p,A_j\right)}{\partial p_k \partial x}}{\frac{\partial D_j\left(p,A_j\right)}{\partial x}}D_j(p,A_j) + \frac{\partial D_j(p,A_j)}{\partial p_k}. \end{aligned}$$

Since $\eta_k(p,x)$ is increasing in x (Assumption 2.ii), we have $\frac{-\frac{\partial^2 D_j(p,x)}{\partial p_k \partial x}}{\frac{\partial D_j(p,x)}{\partial x}} \geq \frac{-\frac{\partial D_j(p,x)}{\partial p_k}}{D_j(p,x)}$, which implies that

$$D2 \geq \frac{-\frac{\partial D_j(p,A_j)}{\partial p_k}}{D_j(p,A_j)} D_j(p,A_j) + \frac{\partial D_j(p,A_j)}{\partial p_k} = 0.$$

This implies that $-\frac{\partial^2 \Pi_j}{\partial p_j \partial p_k}|_{p=p^*} \leq 0$ in assumptions 3.i), 3.ii), 3.iii) and 3.iv). Since $\delta_j > 0$, $\gamma_j \leq 0$ (Appendix A) and $\frac{\partial D_j}{\partial p_k} > 0$, we have $N_{jk}|_{X=X^*} < 0$.

Now, it only remains to show that N is diagonally dominant. In fact, it can be seen that

$$|N_{jj}| - \sum_{k \neq j} |N_{jk}| = -\frac{\partial^2 \Pi_j}{\partial p_j^2} - \sum_{k \neq j} \frac{\partial^2 \Pi_j}{\partial p_j \partial p_k} + \frac{\gamma_j}{\delta_j}\left[\frac{\partial D_j}{\partial p_j} + \sum_{k \neq j} \frac{\partial D_j}{\partial p_k} + \beta_j\right] > -\frac{\partial^2 \Pi_j}{\partial p_j^2} - \sum_{k \neq j} \frac{\partial^2 \Pi_j}{\partial p_j \partial p_k} + \gamma_j(1 - F_j(z_j)).$$

The last inequality holds because $\frac{\partial D_j}{\partial p_j} + \sum_{k \neq j} \frac{\partial D_j}{\partial p_k} < 0$ and $\frac{\beta_j}{\delta_j} < 1$. To simplify notations, let $\Re_j = -\frac{\partial^2 \Pi_j}{\partial p_j^2} - \sum_{k \neq j} \frac{\partial^2 \Pi_j}{\partial p_j \partial p_k} + \gamma_j(1 - F_j(z_j))$. From equations (A.8)-(B.6), $\Re_j$ can be expressed in two ways:

$$\Re_j = \left(p_j + \lambda_j^- - \lambda_j^+\right) \int_{A_j}^{\bar{z}_j} -\left(\frac{\partial^2 D_j(p,x)}{\partial p_j^2} + \sum_{k \neq j} \frac{\partial^2 D_j(p,x)}{\partial p_k p_j}\right) f_j(x) dx - \lambda_j^- \int_{A_j}^{B_j} -\left(\frac{\partial^2 D_j(p,x)}{\partial p_j^2} + \sum_{k \neq j} \frac{\partial^2 D_j(p,x)}{\partial p_k p_j}\right) f_j(x) dx \tag{B.7}$$

$$-\int_{A_j}^{\bar{z}_j} \frac{\partial D_j(p,x)}{\partial p_j} f_j(x) dx - \int_{A_j}^{\bar{z}_j} \left(\frac{\partial D_j(p,x)}{\partial p_j} + \sum_{k \neq j} \frac{\partial D_j(p,x)}{\partial p_k}\right) f_j(x) dx - \left(\frac{\partial D_j\left(p, \bar{z}_j\right)}{\partial p_j} + \sum_{k \neq j} \frac{\partial D_j\left(p, \bar{z}_j\right)}{\partial p_k}\right)\left(1 - F_j\left(\bar{z}_j\right)\right). \tag{B.8}$$

or

$$\begin{aligned} \Re_j \quad = \quad & \int_{A_j}^{\bar{z}_j} \frac{p_j - \lambda_j^+}{p_j} \frac{\frac{\partial \mathcal{E}_j(p,x)}{\partial p_j} + \sum_{k \neq j} \frac{\partial \mathcal{E}_j(p,x)}{\partial p_k}}{\mathcal{E}_j(p,x)} \mathcal{E}_j(p,x) \frac{\partial D_j(p,x)}{\partial x} \left(1 - F_j(x)\right) dx \\ & - \int_{\bar{z}_j}^{B_j} \frac{\lambda_j^-}{p_j} \frac{\frac{\partial \mathcal{E}_j(p,x)}{\partial p_j} + \sum_{k \neq j} \frac{\partial \mathcal{E}_j(p,x)}{\partial p_k}}{\mathcal{E}_j(p,x)} \mathcal{E}_j(p,x) \frac{\partial D_j(p,x)}{\partial x} \left(1 - F_j(x)\right) dx \end{aligned} \tag{B.9}$$

$$+\frac{\lambda_j^+}{p_j^2} \int_{A_j}^{\bar{z}_j} \mathscr{E}_j(p,x) \frac{\partial D_j(p,x)}{\partial x} \left(1 - F_j(x)\right) dx + \frac{\lambda_j^-}{p_j^2} \int_{\bar{z}_j}^{B_j} \mathscr{E}_j(p,x) \frac{\partial D_j(p,x)}{\partial x} \left(1 - F_j(x)\right) dx \tag{B.10}$$

$$\begin{aligned} & - \int_{A_j}^{\bar{z}_j} \frac{\frac{\partial^2 D_j(p,x)}{\partial p_j \partial x} + \sum_{k \neq j} \frac{\partial^2 D_j(p,x)}{\partial p_k \partial x}}{\frac{\partial D_j(p,x)}{\partial x}} \left[1 - \frac{p_j - \lambda_j^+}{p_j} \mathcal{E}_j(p,x)\right] \frac{\partial D_j(p,x)}{\partial x} \left(1 - F_j(x)\right) dx \\ & - \int_{\bar{z}_j}^{B_j} \frac{\frac{\partial^2 D_j(p,x)}{\partial p_j \partial x} + \sum_{k \neq j} \frac{\partial^2 D_j(p,x)}{\partial p_k \partial x}}{\frac{\partial D_j(p,x)}{\partial x}} \frac{\lambda_j^-}{p_j} \mathcal{E}_j(p,x) \frac{\partial D_j(p,x)}{\partial x} \left(1 - F_j(x)\right) dx \\ & - \left(\frac{\partial D_j(p,A_j)}{\partial p_j} + \sum_{k \neq j} \frac{\partial D_j(p,A_j)}{\partial p_k}\right) \end{aligned} \tag{B.11}$$

As in the proof of Theorem 1, the rest of the argument for the sign of $\Re_j$ is divided into two cases.

First case: $\frac{\partial^2 D_j(p,x)}{\partial p_j^2} + \sum_{k \neq j} \frac{\partial^2 D_j(p,x)}{\partial p_k p_j} \leq 0$

In this case, $\Re_j = E1 + E2$, where $E1$ and $E2$ are given by the terms in (B.7) and (B.8), respectively. Because $\frac{\partial D_j(p,x)}{\partial p_j} + \sum_{k \neq j} \frac{\partial D_j(p,x)}{\partial p_k} \leq 0$, term $E2$ is non-negative. If $D_j(p,x)$ satisfies Assumption 3.i), $\frac{\partial^2 D_j(p,x)}{\partial p_j^2} + \sum_{k \neq j} \frac{\partial^2 D_j(p,x)}{\partial p_k p_j}$ is increasing in x and

$$E1 \geq -\left(\frac{\partial^2 D_j\left(p, \bar{z}_j\right)}{\partial p_j^2} + \sum_{k \neq j} \frac{\partial^2 D_j\left(p, \bar{z}_j\right)}{\partial p_k p_j}\right)\left[\left(p_j + \lambda_j^- - \lambda_j^+\right) F_j\left(\bar{z}_j\right) - \lambda_j^-\right] = -\left(\frac{\partial^2 D_j\left(p, \bar{z}_j\right)}{\partial p_j^2} + \sum_{k \neq j} \frac{\partial^2 D_j\left(p, \bar{z}_j\right)}{\partial p_k p_j}\right)\left[p_j - c_j - \rho_j\right] \geq 0.$$

If $D_j(p,x)$ satisfies Assumption 3.ii) or 3.iii), we again mimic the arguments used in the proof of Theorem 1 to show that $E1$ is non-negative.

Second case: $\frac{\partial^2 D_j(p,x)}{\partial p_j^2} + \sum_{k\neq j} \frac{\partial^2 D_j(p,x)}{\partial p_k p_j} \geq 0.$

Based on Assumption 3.iv), $\frac{\partial^2 D_j(p,x)}{\partial p_j \partial x} + \sum_{k\neq j} \frac{\partial^2 D_j(p,x)}{\partial p_k \partial x} \leq 0$, $\frac{\frac{\partial \mathcal{E}_j(p,x)}{\partial p_j} + \sum_{k\neq j}\frac{\partial \mathcal{E}_j(p,x)}{\partial p_k}}{\mathcal{E}_j(p,x)}$ is decreasing in x and $\frac{\frac{\partial^2 D_j(p,x)}{\partial p_j \partial x} + \sum_{k\neq j}\frac{\partial^2 D_j(p,x)}{\partial p_k \partial x}}{\frac{\partial D_j(p,x)}{\partial x}}$ is independent of x. In this case, $\Re_j = F1 + F2 + F3$, where $F1$, $F2$, and $F3$ are given by the terms in (B.9), (B.10), and (B.11), respectively. Because $\frac{\partial D_j(p,x)}{\partial x} \geq 0$, term $F2$ is non-negative. Since $\frac{\frac{\partial \mathcal{E}_j(p,x)}{\partial p_j} + \sum_{k\neq j}\frac{\partial \mathcal{E}_j(p,x)}{\partial p_k}}{\mathcal{E}_j(p,x)}$ is decreasing in x,

$$F1 \geq \frac{\frac{\partial \mathscr{E}_j\left(p,\bar{z}_j\right)}{\partial p_j} + \sum_{k\neq j}\frac{\partial \mathscr{E}_j\left(p,\bar{z}_j\right)}{\partial p_k}}{\mathscr{E}_j\left(p,\bar{z}_j\right)}\left[\int_{A_j}^{\bar{z}_j} \frac{p_j - \lambda_j^+}{p_j}\mathscr{E}_j(p,x)\frac{\partial D_j(p,x)}{\partial x}\left(1 - F_j(x)\right)dx - \int_{\bar{z}_j}^{B_j}\frac{\lambda_j^-}{p_j}\mathscr{E}_j(p,x)\frac{\partial D_j(p,x)}{\partial x}\left(1 - F_j(x)\right)dx\right]$$

$$= \frac{\frac{\partial \mathscr{E}_j\left(p,\bar{z}_j\right)}{\partial p_j} + \sum_{k\neq j}\frac{\partial \mathscr{E}_j\left(p,\bar{z}_j\right)}{\partial p_k}}{\mathscr{E}_j\left(p,\bar{z}_j\right)}\left[D_j(p,A_j) + \int_{A_j}^{\bar{z}_j}\frac{\partial D_j(p,x)}{\partial x}\left(1 - F_j(x)\right)dx\right] \geq 0.$$

The last equality holds because $\frac{\partial \mathcal{E}_j(p,x)}{\partial p_j} + \sum_{k\neq j}\frac{\partial \mathcal{E}_j(p,x)}{\partial p_k} \geq 0$ (Assumption 2.iii) and equation (A.5).

Now, because $\frac{\frac{\partial^2 D_j(p,x)}{\partial p_j \partial x} + \sum_{k\neq j}\frac{\partial^2 D_j(p,x)}{\partial p_k \partial x}}{\frac{\partial D_j(p,x)}{\partial x}}$ is independent of x,

$$\begin{aligned} F3 &= \frac{\frac{\partial^2 D_j(p,A_j)}{\partial p_j \partial x} + \sum_{k\neq j}\frac{\partial^2 D_j(p,A_j)}{\partial p_k \partial x}}{\frac{\partial D_j(p,A_j)}{\partial x}}\left[\int_{A_j}^{\bar{z}_j}\left[\frac{p_j - \lambda_j^+}{p_j}\mathscr{E}_j(p,x) - 1\right]\frac{\partial D_j(p,x)}{\partial x}\left(1 - F_j(x)\right)dx - \int_{\bar{z}_j}^{B_j}\frac{\lambda_j^-}{p_j}\mathscr{E}_j(p,x)\frac{\partial D_j(p,x)}{\partial x}\left(1 - F_j(x)\right)dx\right] \\ &\quad - \left(\frac{\partial D_j(p,A_j)}{\partial p_j} + \sum_{k\neq j}\frac{\partial D_j(p,A_j)}{\partial p_k}\right) \\ &= \frac{\frac{\partial^2 D_j(p,A_j)}{\partial p_j \partial x} + \sum_{k\neq j}\frac{\partial^2 D_j(p,A_j)}{\partial p_k \partial x}}{\frac{\partial D_j(p,A_j)}{\partial x}} D_j(p,A_j) - \left(\frac{\partial D_j(p,A_j)}{\partial p_j} + \sum_{k\neq j}\frac{\partial D_j(p,A_j)}{\partial p_k}\right) \end{aligned}$$

Since $\frac{\frac{\partial \eta_j(p,x)}{\partial x}}{p_j} + \sum_{k\neq j}\frac{\frac{\partial \eta_k(p,x)}{\partial x}}{p_k} \leq 0$ (Assumption 2.ii), we have

$$\frac{\frac{\partial^2 D_j(p,x)}{\partial p_j \partial x} + \sum_{k\neq j}\frac{\partial^2 D_j(p,x)}{\partial p_k \partial x}}{\frac{\partial D_j(p,x)}{\partial x}} \geq \frac{\frac{\partial D_j(p,x)}{\partial p_j} + \sum_{k\neq j}\frac{\partial D_j(p,x)}{\partial p_k}}{D_j(p,x)},$$

which implies that

$$F3 \geq \frac{\frac{\partial D_j(p,A_j)}{\partial p_j} + \sum_{k\neq j}\frac{\partial D_j(p,A_j)}{\partial p_k}}{D_j(p,A_j)} D_j(p,A_j) - \left(\frac{\partial D_j(p,A_j)}{\partial p_j} + \sum_{k\neq j}\frac{\partial D_j(p,A_j)}{\partial p_k}\right) = 0.$$

Consequently, $\Re_j > 0$ for assumptions 3.i), 3.ii), 3.iii) and 3.iv) and matrix N is strictly diagonally dominant with positive diagonal and negative off-diagonal terms, implying that $det(N)|_{X=X^*} > 0$. □

Appendix C. Sensitivity analysis

For simplicity, we examine the special case when $D_j = D_j(p_j, z_j)$. Using the dominance effect among retailers, the proofs can be easily extended to the general case when $D_j = D_j(p, z_j)$.

By definition of z_j, $D_j(p_j^*, z_j^*) = s_j^* = \sum_{i=1}^I q_{ij}^*$, with $z_j^* = F_j^{-1}\left(\frac{p_j^* - c_j - \rho_j^* + \lambda_j^-}{p_j^* + \lambda_j^- - \lambda_j^-}\right)$ and $\rho_j^* = \frac{\partial c_{ij}(q_{ij}^*)}{\partial q_{ij}}$, $\forall i = 1, 2, \cdots I$. Therefore, $\frac{\partial}{\partial q_{lj}}\left(\frac{\partial c_{ij}(q_{ij}^*)}{\partial q_{ij}}\right) = \frac{\partial}{\partial q_{lj}}\left(\frac{\partial c_{lj}(q_{lj}^*)}{\partial q_{lj}}\right)$, $\forall i = 1,$

$2,\cdots I-1$, which is equivalent to $\frac{\partial^2 c_{ij}(q_{ij}^*)}{\partial q_{ij}^2}\frac{dq_{ij}^*}{dq_{Ij}^*} = \frac{\partial^2 c_{Ij}(q_{Ij}^*)}{\partial q_{Ij}^2}$. That implies that $\frac{dq_{ij}^*}{dq_{Ij}^*} = \frac{\frac{\partial^2 c_{Ij}(q_{Ij}^*)}{\partial q_{Ij}^2}}{\frac{\partial^2 c_{ij}(q_{ij}^*)}{\partial q_{ij}^2}}$, $\forall i = 1,2,\cdots I-1$.

C1. Proof of Proposition 1

Note that at the equilibrium, $D_j(p_j^*, z_j^*) = s_j^* = \sum_{i=1}^{I} q_{ij}^*$. Taking the derivative with respect to c_j yields

$$\sum_{i=1}^{I}\frac{dq_{ij}^*}{dc_j} = \frac{\partial D_j\left(p_j^*, z_j^*\right)}{\partial p_j}\frac{dp_j^*}{dc_j} + \frac{\frac{\partial D_j\left(p_j^*, z_j^*\right)}{\partial x}}{(p_j^* + \lambda_j^- - \lambda_j^+)f_j(z_j^*)}\left[-1 - \frac{\partial^2 c_{Ij}\left(q_{Ij}^*\right)}{\partial q_{Ij}^2}\frac{dq_{Ij}^*}{dc_j} + \left(1 - F_j\left(z_j^*\right)\right)\frac{dp_j^*}{dc_j}\right],$$

which implies that $\frac{dq_{Ij}^*}{dc_j}\left[1 + \alpha_j + \frac{\partial^2 c_{Ij}(q_{Ij}^*)}{\partial q_{Ij}^2}\beta_j\right] = \frac{dp_j^*}{dc_j}\gamma_j - \beta_j$, where $\alpha_j = \sum_{i=1}^{I-1}\frac{\frac{\partial^2 c_{Ij}(q_{Ij}^*)}{\partial q_{Ij}^2}}{\frac{\partial^2 c_{ij}(q_{ij}^*)}{\partial q_{ij}^2}} > 0$, $\beta_j = \frac{\frac{\partial D_j(p_j^*, z_j^*)}{\partial x}}{(p_j^* + \lambda_j^- - \lambda_j^+)f_j(z_j^*)} > 0$ and $\gamma_j = \frac{\partial D_j(p_j^*, z_j^*)}{\partial p_j} + \frac{\frac{\partial D_j(p_j^*, z_j^*)}{\partial x}(1 - F_j(z_j^*))}{(p_j^* + \lambda_j^- - \lambda_j^+)f_j(z_j^*)} \le 0$ (Appendix A). Therefore, $\frac{dq_{Ij}^*}{dc_j} = \left\{\frac{dp_j^*}{dc_j}\gamma_j - \beta_j\right\}\Big/\left\{1 + \alpha_j + \frac{\partial^2 c_{Ij}(q_{Ij}^*)}{\partial q_{Ij}^2}\beta_j\right\}$ Next, taking the derivative of the equilibrium equation $\partial\Pi_j/\partial p_j = 0$ with respect to c_j gives $\frac{\partial^2 \Pi_j}{\partial p_j^2}\frac{dp_j^*}{dc_j} + \gamma_j\left[-1 - \frac{\partial^2 c_{Ij}(q_{Ij}^*)}{\partial q_{Ij}^2}\frac{dq_{Ij}^*}{dc_j}\right] = 0$. Using the above and few algebraic manipulations shows that $\frac{dp_j^*}{dc_j} = \{\gamma_j(1 + \alpha_j)\}/\left\{\frac{\partial^2 \Pi_j}{\partial p_j^2}(1 + \alpha_j) + \frac{\partial^2 c_{Ij}(q_{Ij}^*)}{\partial q_{Ij}^2}M_j\right\}$, where $M_j = \frac{\partial^2 \Pi_j}{\partial p_j^2}\beta_j - \gamma_j^2$. From Appendix A, we know that at equilibrium $\frac{\partial^2 \Pi_j}{\partial p_j^2} \le 0$, so $M_j \le 0$ and therefore $\frac{dp_j^*}{dc_j} \ge 0$. To obtain the sign of $\frac{dq_{Ij}^*}{dc_j}$, it can be seen using the formula for $\frac{dp_j^*}{dc_j}$, that the derivative of q_{Ij} with respect to c_j simplifies to

$$\frac{dq_{Ij}^*}{dc_j} = \frac{-M_j}{\frac{\partial^2 \Pi_j}{\partial p_j^2}(1 + \alpha_j) + \frac{\partial^2 c_{Ij}\left(q_{Ij}^*\right)}{\partial q_{Ij}^2}M_j}.$$

Since $M_j \le 0$, $\frac{dq_{Ij}^*}{dc_j} \le 0$ and $\frac{dq_{ij}^*}{dc_j} = \frac{dq_{ij}^*}{dq_{Ij}^*}\frac{dq_{Ij}^*}{dc_j} \le 0, \forall i = 1,2,\cdots I-1$. Note that at equilibrium, $\sum_{n=1}^{N}\tilde{q}_{ni}^* = \sum_{j=1}^{J} q_{ij}^*$ and therefore $\frac{d\tilde{q}_{ni}^*}{dc_j} \le 0, \forall n = 1,2,\cdots N, \forall i = 1,2, \cdots I$.

From the definitions of z_j and ρ_j, we get

$$0 \ge \frac{d\rho_j^*}{dc_j} = \left(\frac{\partial^2 c_{Ij}\left(q_{Ij}^*\right)}{\partial q_{Ij}^2}\right)\frac{dq_{Ij}^*}{dc_j} = \frac{-\left(\frac{\partial^2 c_{Ij}\left(q_{Ij}^*\right)}{\partial q_{Ij}^2}\right)M_j}{\frac{\partial^2 \Pi_j}{\partial p_j^2}(1 + \alpha_j) + \frac{\partial^2 c_{Ij}\left(q_{Ij}^*\right)}{\partial q_{Ij}^2}M_j} \ge \frac{-\left(\frac{\partial^2 c_{Ij}\left(q_{Ij}^*\right)}{\partial q_{Ij}^2}\right)M_j}{\frac{\partial^2 c_{Ij}\left(q_{Ij}^*\right)}{\partial q_{Ij}^2}M_j} = -1,$$

and $\frac{dz_j^*}{dc_j} = \left[(1 - F_j(z_j^*))\frac{dp_j^*}{dc_j} - 1 - \frac{d\rho_j^*}{dc_j}\right]\Big/[(p_j^* + \lambda_j^- - \lambda_j^+)f_j(z_j^*)]$ reduces to

$$\frac{dz_j^*}{dc_j} = (1 + \alpha_j)\left[\left(1 - F_j\left(z_j^*\right)\right)\gamma_j - \frac{\partial^2 \Pi_j}{\partial p_j^2}\right]\Big/\left[\frac{\partial^2 \Pi_j}{\partial p_j^2}(1 + \alpha_j) + \frac{\partial^2 c_{Ij}\left(q_{Ij}^*\right)}{\partial q_{Ij}^2}M_j\right]$$

and has the same sign as $\frac{\partial^2 \Pi_j}{\partial p_j^2} - (1 - F_j(z_j^*))\gamma_j = A1 + A2$, where $A1$ and $A2$ are defined by (A.8) and (A.9) in Appendix A, respectively. It follows that $\frac{dz_j^*}{dc_j} \le 0$, since $A1 + A2 \le 0$, as shown in Appendix A.

For the raw material suppliers, manufacturers, and retailers' profits, it can be seen that $\frac{\partial \Pi_n^S}{\partial c_j}$ follows the same sign of $\frac{\partial \tilde{q}_{ni}^*}{\partial c_j}$, $\frac{\partial \Pi_i^M}{\partial c_j} = \sum_{j=1}^{J}\frac{d\rho_j^*}{dc_j}q_{ij}^* < 0$, and $\frac{\partial \Pi_j}{\partial c_j} = \left(-1 - \frac{d\rho_j^*}{dc_j}\right)\sum_{i=1}^{I} q_{ij}^* < 0$, since $-1 \le \frac{d\rho_j^*}{dc_j} \le 0$, as shown before. Consequently, the expected profits of raw material suppliers, manufacturers, and retailers decrease with c_j, which implies that the total profit Π decreases with c_j.

C2. Proof of Proposition 2

The arguments used in this subsection are quite similar to those presented in subsection C.1, with the only difference beiing that derivatives are taken with respect to λ_j^-. In particular, following the same steps, it can be easily shown that

$\frac{dq_{Ij}^*}{d\lambda_j^-} = \left\{\frac{dp_j^*}{d\lambda_j^-}\gamma_j + \beta_j(1 - F_j(z_j^*))\right\}\Big/\left\{1 + \alpha_j + \frac{\partial^2 c_{Ij}(q_{Ij}^*)}{\partial q_{Ij}^2}\beta_j\right\}$. Computing $\frac{\partial^2 \Pi_j}{\partial \lambda_j^- \partial p_j}$ at the equilibrium and using arguments similiar to those in C.1 yields

$$\frac{dp_j^*}{d\lambda_j^-} = \frac{(1+\alpha_j)\left(\Delta_j^- - \beta_j\left(1-F_j\left(z_j^*\right)\right)^2\right) + \frac{\partial^2 c_{Ij}\left(q_{Ij}^*\right)}{\partial q_{Ij}^2}\beta_j\int_{z_j^*}^{B_j}\frac{\partial D_j\left(p_j^*,x\right)}{\partial p_j}f_j(x)dx}{\frac{\partial^2\Pi_j}{\partial p_j^2}(1+\alpha_j) + \frac{\partial^2 c_{Ij}\left(q_{Ij}^*\right)}{\partial q_{Ij}^2}M_j},$$

where $\Delta_j^- = \int_{z_j^*}^{B_j}\frac{\partial D_j(p_j^*,x)}{\partial p_j}f_j(x)dx - \frac{\partial D_j(p_j^*,z_j^*)}{\partial p_j}(1-F_j(z_j^*))$. Substituting in the above formula for $\frac{dq_{Ij}^*}{d\lambda_j^-}$ yields

$$\frac{dq_{Ij}^*}{d\lambda_j^-} = \frac{M_j\left(1-F_j\left(z_j^*\right)\right) + \gamma_j\int_{z_j^*}^{B_j}\frac{\partial D_j\left(p_j^*,x\right)}{\partial p_j}f_j(x)dx}{\frac{\partial^2\Pi_j}{\partial p_j^2}(1+\alpha_j) + \frac{\partial^2 c_{Ij}\left(q_{Ij}^*\right)}{\partial q_{Ij}^2}M_j}.$$

It can be easily verified that $\frac{d\rho_j^*}{d\lambda_j^-} = \frac{\partial^2 c_{Ij}(q_{Ij}^*)}{\partial q_{Ij}^2}\frac{dq_{Ij}^*}{d\lambda_j^-}$ and that $\frac{dz_j^*}{d\lambda_j^-}$ simplifies to

$$\frac{dz_j^*}{d\lambda_j^-} = \frac{(1+\alpha_j)\left(1-F_j\left(z_j^*\right)\right)\left\{(A1+A2) + \left[1 - \frac{\frac{\partial^2 c_{Ij}\left(q_{Ij}^*\right)}{\partial q_{Ij}^2}\frac{\partial D_j\left(p_j^*,z_j^*\right)}{\partial p_j}}{(1+\alpha_j)\left(1-F_j\left(z_j^*\right)\right)}\right]\int_{z_j^*}^{B_j}\frac{\partial D_j\left(p_j^*,x\right)}{\partial p_j}f_j(x)dx\right\}}{\left[(p_j^* + \lambda_j^- - \lambda_j^+)f_j(z_j^*)\right]\left[\frac{\partial^2\Pi_j}{\partial p_j^2}(1+\alpha_j) + \frac{\partial^2 c_{Ij}\left(q_{Ij}^*\right)}{\partial q_{Ij}^2}M_j\right]}.$$

Using the results of Appendix A, it can be seen that $\frac{dz_j^*}{d\lambda_j^-} \geq 0$ for any demand model. The signs of $\frac{dp_j^*}{d\lambda_j^-}$ and $\frac{dq_{Ij}^*}{d\lambda_j^-}$ depend in general on the demand model. In particular, if $\frac{\partial D_j(p_j,x)}{\partial p_j}$ is increasing in x, then it is easy to show that $\frac{\partial^2\Pi_j}{\partial p_j^2} - \gamma_j(1-F_j(z_j^*)) = A1 + A2 \leq 0$ implying that $\gamma_j\int_{z_j^*}^{B_j}\frac{\partial D_j(p_j^*,x)}{\partial p_j}f_j(x)dx + M_j(1 - F_j(z_j^*)) \leq (1-F_j(z_j^*))\beta_j\left[\frac{\partial^2\Pi_j}{\partial p_j^2} - \gamma_j(1-F_j(z_j^*))\right] \leq 0$. Therefore, $\frac{dq_{Ij}^*}{d\lambda_j^-} \geq 0$ but the sign of $\frac{dp_j^*}{d\lambda_j^-}$ depends on the model parameters. However, if $\frac{\partial D_j(p_j,x)}{\partial p_j}$ is decreasing in x, then $\Delta_j^- \leq 0$ implying that $\frac{dp_j^*}{d\lambda_j^-} \geq 0$ and the sign of $\frac{dq_{Ij}^*}{d\lambda_j^-}$ depends on the model parameters. In both cases, $\frac{d\bar{q}_{ni}^*}{d\lambda_j^-}$ $(1 \leq n \leq N,\ 1 \leq i \leq I)$, $\frac{dq_{ij}^*}{d\lambda_j^-}$ $(1 \leq i \leq I-1)$, and $\frac{d\rho_j^*}{d\lambda_j^-}$ have the same sign as $\frac{dq_{Ij}^*}{d\lambda_j^-}$.

For the expected profits of raw material suppliers and manufacturers, it can be seen that $\frac{\partial\Pi_n^S}{\partial\lambda_j^-}$ have the same sign as $\frac{d\bar{q}_{ni}^*}{d\lambda_j^-}$ and $\frac{\partial\Pi_i^M}{\partial\lambda_j^-} = \sum_{j=1}^J\frac{d\rho_j^*}{d\lambda_j^-}q_{ij}^*$ which has the same sign as $\frac{dq_{ij}^*}{d\lambda_j^-}$. On the other hand, $\frac{\partial\Pi_j}{\partial\lambda_j^-} = -\Theta_j(p^*, z_j^*) - \frac{d\rho_j^*}{d\lambda_j^-}\sum_{i=1}^I q_{ij}^*$, which depends on the sign of $\frac{d\rho_j^*}{d\lambda_j^-}$. Summing the above yields, $\frac{\partial\Pi}{\partial\lambda_j^-} = -\Theta_j(p^*, z_j^*) < 0$, implying that the total profit Π decreases with λ_j^-.

C3. *Proof of Proposition 3*

Again, using similar arguments to those in C.1, it can be verified that

$$\frac{dq_{Ij}^*}{d\lambda_j^+} = \left\{\frac{dp_j^*}{d\lambda_j^+}\gamma_j + \beta_j F_j\left(z_j^*\right)\right\}\Big/\left\{1+\alpha_j+\frac{\partial^2 c_{Ij}\left(q_{Ij}^*\right)}{\partial q_{Ij}^2}\beta_j\right\},$$

and using $\frac{\partial^2\Pi_j}{\partial\lambda_j^+\partial p_j}$, we get

$$\frac{dp_j^*}{d\lambda_j^+} = \frac{(1+\alpha_j)\left(\Delta_j^+ - \beta_j F_j\left(z_j^*\right)\left(1-F_j\left(z_j^*\right)\right)\right) + \frac{\partial^2 c_{Ij}\left(q_{Ij}^*\right)}{\partial q_{Ij}^2}\beta_j\int_{A_j}^{z_j^*}\frac{\partial D_j\left(p_j^*,x\right)}{\partial p_j}f_j(x)dx}{\frac{\partial^2\Pi_j}{\partial p_j^2}(1+\alpha_j) + \frac{\partial^2 c_{Ij}\left(q_{Ij}^*\right)}{\partial q_{Ij}^2}M_j},$$

where $\Delta_j^+ = \int_{A_j}^{z_j^*}\frac{\partial D_j(p_j^*,x)}{\partial p_j}f_j(x)dx - \frac{\partial D_j(p_j^*,z_j^*)}{\partial p_j}F_j(z_j^*)$. Substituting in the above formula for $\frac{dq_{Ij}^*}{d\lambda_j^+}$ gives

$$\frac{dq_{Ij}^*}{d\lambda_j^+} = \frac{M_j F_j\left(z_j^*\right) + \gamma_j\int_{A_j}^{z_j^*}\frac{\partial D_j\left(p_j^*,x\right)}{\partial p_j}f_j(x)dx}{\frac{\partial^2\Pi_j}{\partial p_j^2}(1+\alpha_j) + \frac{\partial^2 c_{Ij}\left(q_{Ij}^*\right)}{\partial q_{Ij}^2}M_j}$$

It is also easy to see that $\frac{d\rho_j^*}{d\lambda_j^+} = \frac{\partial^2 c_{Ij}(q_{Ij}^*)}{\partial q_{Ij}^2}\frac{dq_{Ij}^*}{d\lambda_j^+}$ and that $\frac{dz_j^*}{d\lambda_j^+}$ is given by

$$\frac{dz_j^*}{d\lambda_j^+} = \frac{(1+\alpha_j)F_j\left(z_j^*\right)(A1+A2) + \left[1 - F_j\left(z_j^*\right) - \frac{\partial^2 c_{Ij}\left(q_{Ij}^*\right)}{\partial q_{Ij}^2}\frac{\partial D_j\left(p_j^*,z_j^*\right)}{\partial p_j}\right]\int_{A_j}^{z_j^*}\frac{\partial D_j\left(p_j^*,x\right)}{\partial p_j}f_j(x)dx}{(p_j^*+\lambda_j^- - \lambda_j^+)f_j\left(z_j^*\right)\left[\frac{\partial^2\Pi_j}{\partial p_j^2}(1+\alpha_j) + \frac{\partial^2 c_{Ij}\left(q_{Ij}^*\right)}{\partial q_{Ij}^2}M_j\right]}.$$

Clearly $\frac{dz_j^*}{d\lambda_j^+} \geq 0$ for all models but the signs of $\frac{dp_j^*}{d\lambda_j^+}$ and $\frac{dq_{Ij}^*}{d\lambda_j^+}$ depend on the demand model. In fact, when $\frac{\partial D_j(p_j,x)}{\partial p_j}$ is increasing in x, then $\Delta_j^+ \leq 0$. Therefore, $\frac{dp_j^*}{d\lambda_j^+} \geq 0$ and the sign of $\frac{dq_{Ij}^*}{d\lambda_j^+}$ depends on the model parameters. However, when $\frac{\partial D_j(p,x)}{\partial p_j}$ is decreasing in x, it is easy to prove that $\gamma_j \int_{A_j}^{z_j^*}\frac{\partial D_j(p_j^*,x)}{\partial p_j}f_j(x)dx + M_jF_j(z_j^*) \leq 0$. Therefore, $\frac{dq_{Ij}^*}{d\lambda_j^+} \geq 0$ and the sign of $\frac{dp_j^*}{d\lambda_j^+}$ depends on the model parameters. As in the previous subsection, $\frac{d\bar{q}_{ni}^*}{d\lambda_j^+}(1 \leq n \leq N, 1 \leq i \leq I)$, $\frac{dq_{ij}^*}{d\lambda_j^+}(1 \leq i \leq I-1)$, $\frac{d\rho_j^*}{d\lambda_j^+}$, $\frac{\partial \Pi_n^S}{\partial \lambda_j^+}$, and $\frac{\partial \Pi_i^M}{d\lambda_j^+}$ have the same sign as $\frac{dq_{Ij}^*}{d\lambda_j^+}$.

For the expected profits of the retailers, it can be seen that $\frac{\partial \Pi_j}{\partial \lambda_j^+} = \Lambda_j(p^*, z_j^*) - \frac{\partial \rho_j^*}{\partial \lambda_j^+}\sum_{i=1}^I q_{ij}^*$, which can be positive or negative depending on the model parameters. Summing all expected profits yields, $\frac{\partial \Pi}{\partial \lambda_j^+} = \Lambda_j(p^*, z_j^*) > 0$, implying that the total profit Π increases with λ_j^+.

C4. Proof of Proposition 4

To proof Proposition 4, we use the mixed linear-exponential model $D_j(p_j, x) = \mu_j(p_j) + x\sigma_j(p)$ with $\mu_j(p) = a_j - b_jp_j$ and examine the effect of varying a_j. Note that the parameter a_j is used to control the demand level and a change in a_j results in a change of the demand level without changing demand variablility and demand dependence on p. Similar analysis could be carried out for parameters b_j and c_{jk}. Repeating steps similar to those in the above subsections, we get $\frac{dq_{Ij}^*}{da_j} = \left(\frac{dp_j^*}{da_j}\gamma_j + 1\right)\Big/\left(1 + \alpha_j + \frac{\partial^2 c_{Ij}(q_{Ij}^*)}{\partial q_{Ij}^2}\beta_j\right)$. Using the equilibrium equations and adapting the arguments of the previous sections yields

$$\frac{dp_j^*}{da_j} = \frac{-(1+\alpha_j) + (\gamma_j - \beta_j)\frac{\partial^2 c_{Ij}\left(q_{Ij}^*\right)}{\partial q_{Ij}^2}}{\frac{\partial^2\Pi_j}{\partial p_j^2}(1+\alpha_j) + \frac{\partial^2 c_{Ij}\left(q_{Ij}^*\right)}{\partial q_{Ij}^2}M_j},$$

and

$$\frac{dq_{Ij}^*}{da_j} = \frac{\frac{\partial^2\Pi_j}{\partial p_j^2} - \gamma_j}{\frac{\partial^2\Pi_j}{\partial p_j^2}(1+\alpha_j) + \frac{\partial^2 c_{Ij}\left(q_{Ij}^*\right)}{\partial q_{Ij}^2}M_j}.$$

The term $\gamma_j - \beta_j = \frac{\partial D_j(p_j^*,z_j^*)}{\partial p_j} - \frac{\frac{\partial D_j(p_j^*,z_j^*)}{dx}F_j(z_j^*)}{(p_j^*+\lambda_j^- - \lambda_j^+)f_j(z_j^*)} \leq 0$. Moreover, it is easy to show that at equilibrium $\frac{\partial^2\Pi_j}{\partial p_j^2} - \gamma_j \leq 0$ for the mixed linear-exponential model, implying that $\frac{dp_j^*}{da_j} \geq 0$ and $\frac{dq_{Ij}^*}{da_j} \geq 0$. Consequently, $\frac{d\rho_j^*}{da_j} \geq 0$, $\frac{dq_{ij}^*}{da_j} = \frac{dq_{ij}^*}{dq_{Ij}^*}\frac{dq_{Ij}^*}{da_j} \geq 0, \forall i = 1, 2, \cdots I-1$, and $\frac{d\bar{q}_{ni}^*}{da_j} > 0$.

For the total expected profit, it can be seen that $\frac{\partial \Pi}{\partial a_j} = (p_j^* + \lambda_j^- - \lambda_j^+)F_j(z_j^*) - \lambda_j^- > 0$ implying that the total profit Π increases with a_j. The expected profits of raw material suppliers and manufacturers increase with a_j since $\frac{\partial \bar{q}_{ni}^*}{\partial a_j} > 0$ and $\frac{\partial \rho_j^*}{\partial a_j} > 0$.

C5. Proof of Proposition 5

To proof Proposition 5, we use the linear demand function $D_j(p_j, x) = a_j - b_jp_j + m_jx$ and study the effect of varying m_j. Note that for this model, a change in m_j results in a change of demand variability without changing demand level or demand dependence on p. Mimicking the same steps as in the previous subsections yields

$$\frac{dp_j^*}{dm_j} = \frac{\Theta_j\left(z_j^*\right)(1+\alpha_j) + \left(\gamma_j z_j^* + \Theta_j\left(z_j^*\right)\beta_j\right)\frac{\partial^2 c_{Ij}\left(q_{Ij}^*\right)}{\partial q_{Ij}^2}}{\frac{\partial^2\Pi_j}{\partial p_j^2}(1+\alpha_j) + \frac{\partial^2 c_{Ij}\left(q_{Ij}^*\right)}{\partial q_{Ij}^2}M_j}$$

and

$$\frac{dq_{Ij}^*}{dm_j} = \frac{z_j^*\frac{\partial^2\Pi_j}{\partial p_j^2} + \gamma_j\Theta_j\left(z_j^*\right)}{\frac{\partial^2\Pi_j}{\partial p_j^2}(1+\alpha_j) + \frac{\partial^2 c_{Ij}\left(q_{Ij}^*\right)}{\partial q_{Ij}^2}M_j},$$

where $\Theta_j(z_j^*) = \int_{z_j^*}^{B_j}(x - z_j^*)f_j(x)dx = -\int_{A_j}^{z_j^*}xf_j(x)dx - z_j^*(1 - F_j(z_j^*))$.

The signs of $\frac{dp_j^*}{dm_j}$ and $\frac{dq_{Ij}^*}{dm_j}$ depend on the sign of z_j. In fact, if $z_j^* \leq 0$, then $\Theta_j(z_j^*)(1+\alpha_j) + (\gamma_j z_j^* + \Theta_j(z_j^*)\beta_j)\frac{\partial^2 c_{Ij}(q_{Ij}^*)}{\partial q_{Ij}^2} \geq 0$, implying that $\frac{dp_j^*}{dm_j} \leq 0$. Calculations show that the sign of $\frac{dq_{lj}^*}{dm_j}$ depends on the model parameters. If $z_j^* \geq 0$, then $z_j^* \frac{\partial^2 \Pi_j}{\partial p_j^2} + \gamma_j \Theta_j(z_j^*) \leq 0$, implying that $\frac{dq_{lj}^*}{dm_j} \geq 0$. For $\frac{dp_j^*}{dm_j}$, its sign depends on the model parameters.

For the expected profits, straightforward computation shows that $\frac{\partial \Pi}{\partial m_j} = (p_j^* + \lambda_j^- - \lambda_j^+) \int_{A_j}^{z_j^*} x f_j(x) < 0$ implying that the total profit Π decreases with m_j. For the expected profits of raw material suppliers, manufacturers, and retailers, it can be seen that $\frac{\partial \Pi_n^S}{\partial m_j}$, and $\frac{\partial \Pi_i^M}{\partial m_j}$ have the same sign as $\frac{dq_{ij}^*}{dm_j}$ and the sign of $\frac{\partial \Pi_j}{\partial m_j}$ depends on the model parameters.

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