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Supply chain coordination in a dual sourcing system under the Tailored Base-Surge policy

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ABSTRACT

In this paper, we study the coordination of a dual sourcing supply chain comprising a buyer and two suppliers: a regular and an expedited one. The suppliers differ in lead time and cost, with the expedited supplier offering a shorter lead time at a higher cost than the regular supplier. The buyer uses the Tailored Base-Surge inventory policy, ordering every period a fixed quantity from the regular supplier and using the expedited supplier to meet any excess demand. We employ a novel perspective by assuming that each of the three firms is an independent party optimizing its profit function. We consider two scenarios: in the first scenario, the expedited supplier acts as a spot market, resulting in a two-players game between the buyer and regular supplier. The second scenario considers a three-players game. We derive the conditions for coordination for both scenarios, which we refer to as single and double coordination, and explore the impact of various parameters on the games and coordination. Our findings reveal that in the two-players scenario, regular orders increase with the spot market price, with a greater increase under coordination. Meanwhile, in the three-players scenario, equilibrium can only be sustained by increasing the order quantity from the expedited supplier in case its sourcing cost increases. Moreover, the regular supplier has an incentive to raise its price, whereas the expedited supplier charges a fixed price to the buyer. However, coordination results in the buyer placing fewer expedited orders. We demonstrate that the regular supplier sets its price just below the expedited supplier's price. In contrast, the expedited supplier acts more aggressively in setting its price to either eliminate the regular supplier or charge the maximum possible price.

1. Introduction

Establishing a resilient supply chain has attracted the attention from scientists and practitioners to meet changing market needs and achieve rapid recovery from disruptions, such as the pandemic. An important strategy to establish a resilient supply chain is to rely on at least two different sourcing options (Chowdhury et al., 2021). Recently, McKinsey conducted a study where data were collected from 113 global supply chain leaders from a broad range of industries, revealing that "81% of the respondents say that they have implemented dual-sourcing strategies during the past year [2021], up from 55 percent in 2020" (Alicke et al., 2022). Another recent study shows that 92% of manufacturing firms in the UAE apply a dual- or multi-sourcing strategy to supply critical products (Hamdouch et al., 2023).

In a dual-sourcing system, a buyer facing uncertain demand can source the same product from two different suppliers (or from the same supplier with two different transportation modes). The first, a low-cost offshore supplier is referred to as the *regular supplier*. The other, a responsive onshore supplier is referred to as the *expedited supplier*. The dual sourcing strategy helps firms achieve responsiveness and efficiency (Boute & Van Mieghem, 2015). It can reduce procurement costs, mitigate disruption risks, and guarantee customer satisfaction (Gupta & Ivanov, 2020; Iakovou et al., 2010; Jakšič, 2016; Lyon, 2006). Therefore, dual sourcing systems have received considerable attention in the literature (Svoboda et al., 2021).

Several studies in the literature have shown that such dual sourcing practices are widespread in industry, especially since many companies have come to realize that a sourcing strategy with two (or more simultaneous) suppliers may be more effective than one that relies on a single supplier. For example, Rao et al. (2000) reported on Caterpillar's dual sourcing strategy for compact work tools, Beyer and Ward (2002) on Hewlett-Packard's dual sourcing strategy for its manufacturing plants,

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and Allon and Van Mieghem (2010) on a US-based $10 billion manufacturer of wireless transmission components with two assembly plants: one in China and another in Mexico.

The optimal policy structure for most dual sourcing inventory systems is complex or unknown (Whittemore & Saunders, 1977); therefore, several (heuristic) policies have been proposed and investigated in the literature (Sun & Van Mieghem, 2019; Svoboda et al., 2021). An appealing and practical policy is the Tailored Base-Surge (TBS) policy. Under the TBS policy, the buyer places every period a fixed order quantity at the regular supplier and fulfills excess demand via the expedited supplier (Svoboda et al., 2021). This study proposes a stochastic dual sourcing inventory management system under the TBS policy.

Janakiraman et al. (2015) establishes the optimality of the TBS policy in the special case when the demand has a two-point support distribution. For general demand distributions, Xin and Goldberg (2018) prove the asymptotic optimality of this policy, i.e., when the lead time grows large. From a practical point of view, several cases have been documented where the TBS policy appeared to be an interesting policy in a practical setting. For example, Mini-Case 6 in Van Mieghem and Allon (2008) presents a case study of a large car manufacturer that motivates the usage of the TBS policy. Another case study has been presented by Radstok (2013), in which the TBS policy has been applied to a transportation problem at an international Fast Moving Consumer Goods manufacturer. Xin et al. (2017) applied the TBS policy to a real-world supply chain problem at Walmart.com and showed that the TBS policy resulted in a significant reduction in inventory costs. In general, the presumption is that the regular supplier cannot rapidly change volumes because of long lead times or due to an inflexible production process (Boulaksil & Fransoo, 2010). The benefits of the TBS policy are that it is simple to administer and it eliminates the need to explicitly account for the long lead time.

However, most studies in the literature on dual-sourcing systems consider only the buyer perspective, i.e., a single-echelon setting (Boulaksil et al., 2021; Svoboda et al., 2021). The few studies that consider a multi-echelon setting assume a single, central decision-making authority with access to all relevant information that releases decisions by optimizing a single overall objective function (De Kok et al., 2018). This can only happen when a multinational company owns several sites that supply each other. In reality, decision-making authority in a supply chain is typically distributed over multiple firms, resulting in a decentralized supply chain. Consequently, each firm aims to optimize its objective function, which may conflict with those of other firms (Ma et al., 2013; Van Der Rhee et al., 2010), leading to suboptimal solutions (Lee & Whang, 1999). For instance, if an individual entity wants to secure more profit by charging a price exceeding its marginal cost, the final product price may be costly, consequently affecting sales. This phenomenon, known as double marginalization, negatively affects total supply chain profit. Double marginalization arises because each entity acts independently to secure the largest possible proportion of the total profit, resulting in competition among the supply chain entities (Dellarocas, 2012).

Coordination can reduce the impact of supply chain decentralization and, consequently, that of double marginalization (Cachon, 2003). Supply chain coordination aims to enhance the performance of a decentralized supply chain without changing the decision-making authorities and network structure. The ultimate goal of coordination is to achieve performance similar to that of a centralized network by setting the correct incentives for all entities (Huang et al., 2015; Van Der Rhee et al., 2010). The main mechanism to achieve coordination is contracting. Contracts define transaction rules among entities, providing incentives for sharing risks or rewards (Tsay et al., 1999). They promote coherent decisions and, consequently, efficient management among entities (Ma et al., 2013). Designing an acceptable contract (resulting in viable coordination) for all entities within the supply chain requires that the contract provides a win-win situation resulting in each entity's profit (or benefits) being higher than (or at least equal to) the one from decentralization (Giannoccaro & Pontrandolfo, 2004; Van Der Rhee et al., 2010). Supply chain contracts typically consider trade parameters such as pricing, quantity, delivery, and quality.

In this study, we assume that each firm in the dual sourcing system (the buyer and the two suppliers) is an independent entity with its own objective function that is not (necessarily) aligned with the others. We consider two scenarios. In the first scenario, the expedited supplier is assumed to act as a spot market, and a two-players game between the buyer and regular supplier is analyzed. Then, we provide the conditions allowing for coordination between the two-players, which we refer to as the *single-coordination* case. In the second scenario, we expand upon the first scenario by analyzing a three-players' game and provide the conditions allowing for coordination between the three-players, which we refer to as the *double coordination* case. In this manner, we study supply chain coordination in a dual sourcing system under the TBS policy.

This paper makes two major contributions to the literature. First, we analytically derive coordination conditions for the two- and three-players scenarios. Specifically, we prove that any contracts satisfying the derived conditions will achieve coordination. To illustrate our findings, we use a quantity discount contract as an example of a feasible contract. Second, we analytically and numerically study the impact of several model parameters on the system performance for the game setting and for coordination.

The insights from this study are helpful in better understanding the game dynamics. For example, our analysis shows that in the three-players scenario, the regular supplier acts differently from the expedited supplier. While the regular supplier tries to set its price to just under the expedited supplier's price, the expedited supplier will either set its price low enough to eliminate the regular supplier or charge the maximum possible price for an equilibrium to be achieved. Overall, this study provides valuable insights into two- and three-players' games and how coordination can be achieved within a dual sourcing system.

This paper is organized as follows. In Section 2, we review the literature on dual-sourcing, coordination within dual-sourcing environment, and the use of contracts to achieve coordination. Section 3 describes the studied problem in more detail. A formulation of the Stackelberg games and coordination for the two- and three-players' scenarios are presented in Sections 4 and 5. Section 6 provides the analytical and numerical results. In Section 7, we discuss the main managerial insights, and finally, Section 8 concludes the paper.

2. Literature review

We review the relevant research streams related to this study. In Section 2.1, we briefly review papers related to dual sourcing inventory management with a focus on the TBS policy. In Section 2.2, we review papers on supply chain coordination with dual- or multi-sourcing. Finally, in Section 2.3, we focus on studies that have used quantity-discount contracts to coordinate supply chains.

2.1. Dual sourcing & Tailored Base-Surge (TBS) policy

Dual sourcing inventory management has been studied since the 1960s (Barankin, 1961; Fukuda, 1964; Sapra, 2017). As the optimal policy for the general dual sourcing setting is complex or unknown (Whittemore & Saunders, 1977), several heuristic policies have been proposed and studied in the literature. Examples of such policies include the dual-index (Veeraraghavan & Scheller-Wolf, 2008), order splitting (Sajadieh & Eshghi, 2009), TBS (Allon & Van Mieghem, 2010; Boulaksil et al., 2021; Janssen & de Kok, 1999; Xin & Goldberg, 2018), order smoothing (Boute & Van Mieghem, 2015), dual-index with batch ordering (Wu et al., 2019), and three-index policies (Arts & Kiesmüller, 2013).

The TBS policy is one in which the regular supplier delivers a constant quantity each period, also referred to as the standing order quantity. In contrast, the expedited supplier is controlled by a more flexible policy (Janssen & de Kok, 1999; Rosenshine & Obee, 1976). It combines a push approach (a fixed quantity supplied by the regular supplier) with a pull approach (variable quantities ordered from the expedited supplier). From a practical perspective, it is an attractive policy, especially for a regular supplier who can operate very efficiently. However, the expedited supplier plays a more important role in the buyer maintaining a high service level. Xin and Goldberg (2018) found that the TBS policy is asymptotically optimal as the lead time difference grows. Boulaksil et al. (2021) found that TBS gives satisfactory results when the upstream stage of the supply chain is considered.

A number of studies have considered various aspects, such as deteriorating items (Yu, 2007), recovered items (Mitra, 2009), nonstationary demand Boute et al. (2022), Firoozi et al. (2020), demand forecast updating (Song et al., 2014), an unreliable supplier (Silbermayr & Minner, 2016; Yu et al., 2009), product returns (Janakiraman & Seshadri, 2017), partial observability (Yee et al., 2023), stock-out dependent substitution (Hekimoğlu & Scheller-Wolf, 2023) and behavioral aspects (Xue et al., 2022). A recent comprehensive and excellent review on dual-sourcing inventory models is available in Svoboda et al. (2021).

2.2. Supply chain coordination

Decentralized systems require coordination to reduce the effect of double marginalization and achieve higher profits (Cachon, 2003). Several papers have studied coordinating the buyer–supplier relationship (Cachon & Lariviere, 2005; Disney et al., 2008; Kunter, 2012; Segal & Whinston, 2002; Taylor & Plambeck, 2007). Building resilient supply chains is becoming increasingly important; many firms are implementing dual sourcing strategies. Thus, supply chain coordination with more than one sourcing option is an increasingly relevant research area.

In a multi-sourcing, multi-echelon configuration, Hua et al. (2010) studied a dual-channel supply chain network, where a manufacturer sells a product to a retailer or directly to customers. They considered two supply chain configurations: a centralized configuration, where the manufacturer controls the retailer and direct channel price as well as quoted lead time, and a decentralized configuration managed using the Stackelberg game. They showed that profits and pricing strategies depend greatly on the delivery lead time. Tang and Kouvelis (2011) modeled an order-quantity game followed by an output-to-market game to coordinate a multi-buyers multi-suppliers supply chain under yield uncertainty. They considered the cases of symmetric (in their cost structure and yield distribution) and asymmetric suppliers, where higher yield is compensated with higher wholesale prices. They found a decrease in profit as the suppliers' correlation increases, leading to more correlated buyers' outputs. In addition, Shu et al. (2015) studied the coordination in a network of two manufacturers, one distributor, and one retailer using options and buyback contracts. One manufacturer is unreliable and less expensive, while the other is more reliable and expensive. The reliable manufacturer offers an option contract to the distributor, who offers a buyback contract to the retailer. Their simulation results showed that disruption risks do not affect the quantities ordered from unstable manufacturers offering low prices; however, the quantities ordered from stable manufacturers offering option contracts increase. Luo et al. (2015) considered a two-stage supply chain where the manufacturer procures products from a supplier using a real-option contract or the spot market with price and supply risks. They showed that integrating a real-option contract with the spot market provides more profit and better coordination and that the spot market risks benefit the supplier. Ke et al. (2017) studied supply chain pricing in a manufacturer's two-echelon system and the two-retailers problem under demand and cost uncertainties. They developed three game-theoretical models based on the dominant party, finding that a reduction in supply chain profit and an increase in sale prices occur in the presence of dominant powers in the system. Lan et al. (2018) studied a dual sourcing dual channel system where a manufacturer sells a single product to a retailer using two distribution channels: two suppliers. The two suppliers have different wholesale prices and abilities to absorb unsold inventory. They showed that while single supplier systems can be managed using contracts, dual sourcing systems can be managed through competition; further, the manufacturer is the designer of the structure. Xu et al. (2023) coordinated a dual channel supply chain while considering blockchain, cross-channel effect and logistics services.

2.3. Quantity discounts in coordinating supply chains

Quantity discount contracts coordinate supply chains by giving the buyer an incentive to purchase large quantities to receive reduced unit prices. Meanwhile, the supplier benefits from larger order quantities and possibly reduced fixed and inventory costs. Various papers have studied these benefits in coordinating supply chains under different conditions. For instance, Huang et al. (2011) used quantity discount contracts to encourage retailers to reduce false returns. They showed that this mechanism resolves the profit conflict in the closed-loop supply chain. Yin et al. (2015) studied coordinating a one-buyer and multiple-suppliers problem in an uncertain demand environment using a noncooperative game. They determined the optimal quantity discount policy that helps to develop production, pricing, and inventory plans. They conducted several numerical experiments and showed that profits do not always decline if the mean value of demand decreases. Further, if the manufacturer increases the opportunity loss cost, profit does not always increase. Nie and Du (2017) reported that quantity discount contracts with price ranges dependent on the supplier's wholesale price are insufficient to coordinate a supply chain of one supplier and two retailers under fairness considerations. Moreover, they are inadequate when the demand is a decreasing function of the retail price. Thus, they proposed a quantity discount with a fixed fee contract that enables suppliers to sacrifice part of the profit to motivate retailers to reduce prices, consequently increasing whole-channel profit. Liu et al. (2018) proposed a quantity discount coordinated replenishment and delivery strategy. They showed that although quantity discount enhanced the coordinated replenishment and delivery strategy, resource constraints reduced the effect of quantity discounts in large-sized problems. In addition, coordination constraints were more sensitive than delivery constraints.

2.4. Literature summary and gap

Table 1 presents a comparison of relevant works based on various criteria: supply chain structure (the number of buyers and suppliers), suppliers' cost structure (whether similar or different), and lead time consideration (not mentioned in the model, zero lead time, or positive lead time). The table also outlines the coordination methods used, the types of uncertainties considered, and the inventory management policies employed. The latter is a focal point in our study. Our analysis of the literature reveals a spectrum of supply chain structures explored, ranging from a single supplier and buyer to multiple suppliers and buyers. Notably, the majority of studies focus on single suppliers when addressing coordination problems. Demand uncertainty emerges as a recurring theme in several models (Cachon & Kök, 2010; Giri & Sarker, 2019; Ha & Tong, 2008; Lan et al., 2018; Liu et al., 2017; Mohebbi & Li, 2015; Shu et al., 2015; Taylor & Plambeck, 2007; Yin et al., 2015). A limited number of works have incorporated inventory management, either through the economic order quantity or order-up-to policies (Cachon & Kök, 2010; Disney et al., 2008; Kerkkamp et al., 2018; Liu et al., 2018; Yin et al., 2015). Some also consider positive lead times (Disney et al., 2008; Mohebbi & Li, 2015). However, a significant portion of past research has overlooked lead time and inventory management policy

Table 1
Literature summary.

Paper	SC structure		Suppliers' structure		Lead time			Coordination method	Uncertainty	Inventory management
	NB	NS	S	D	NM	0	> 0			
Segal and Whinston (2002)	1	1	✓		✓			Contracting		
Ha et al. (2003)	1	2		✓	✓			Noncooperative game theory		Economic order quantity
Giannoccaro and Pontrandolfo (2004)	1	1			✓			Revenue sharing contract	De	
Cachon and Lariviere (2005)	N[a]	1	✓		✓			Revenue sharing contract	De	
Battaglini (2007)	1	1		✓	✓			Contracting	Pr	
Taylor and Plambeck (2007)	1	1	✓			✓		Optimal relational contract and Nash bargaining	De	
Ha and Tong (2008)	1	1			✓			Revenue sharing contract and non-cooperative game	De	
Dang (2008)	1	1	✓		✓			Bargaining game		
Disney et al. (2008)	1	1	✓				✓	global optimization	De	order-up-to policy
Zhou and Feng (2010)	1	2		✓		✓		Buy-back contract, wholesale price contract and Stackelberg game model		
Cachon and Kök (2010)	1	2		✓	✓			Contracts	De	Base-stock/Economic order quantity
Van Der Rhee et al. (2010)	1	1		✓	✓			Spanning revenue contract	De	
Hou et al. (2010)	1	2		✓	✓			Buyback contract	De/Yi	
Tang and Kouvelis (2011)	N	N	✓[b]	✓	✓			Quantity order game followed by output-to-market game	Yi	
Huang et al. (2011)	1	1	✓		✓			Quantity discount contract	O[c]	
He and Zhao (2012)	1	1	✓		✓			Wholesale price and buyback contracts	De/Yi	
Kunter (2012)	1	1	✓		✓			Revenue sharing contract and Nash bargaining Stackelberg game		
Qiang et al. (2013)	N	N		✓	✓			Finite-dimensional variational inequality	De/Yi	
Yang et al. (2013)	1	1	✓		✓			Stackelberg game model		
Ma et al. (2013)	1	1	✓		✓			Innovative two-part tariff contract		
Wu et al. (2014)	1	1	✓		✓			Capacity reservation contracts		
Brusset and Agrell (2015)	1	2	✓		✓			Contracting and Stackelberg game model	O[d]	
Mohebbi and Li (2015)	1	N		✓			✓	Dynamic coalitional game theory solved using cooperation algorithm	De/Yi	
Shu et al. (2015)	1	2		✓	✓			Combined options and back contract	De	
Yin et al. (2015)	1	N		✓	✓			Noncooperative game with quantity discount contract	De	Economic lot-sizing policy
Liu et al. (2017)	2	1			✓			Revenue sharing contract	De	
Nie and Du (2017)	2	1		✓	✓			Quantity discount contract	O[e]	
Lan et al. (2018)	1	2		✓	✓			Stackelberg game model	De	
Kerkkamp et al. (2018)	1	1	✓			✓		Contracting solved using KKT	O[f]	Economic order quantity
Liu et al. (2018)	N	N		✓	✓			Mathematical modeling using hybrid Tabu Search		Economic order quantity
Giri and Sarker (2019)	1	1	✓		✓			Pairwise and spanning revenue sharing contract	De/Yi	
Sarkar and Bhala (2021)	1	1	✓		✓			Constant wholesale price contract		
Xu et al. (2023)	2	2		✓	✓			Co-opetition		
This work	1	2		✓			✓	Stackelberg game and quantity discount contract	De	TBS policy

NB = Number of buyers, NS = number of suppliers, S = similar, D = different, NM = not mentioned, De = demand, Pr = price, Yi = yield, O = other.

[a] They also investigated the case of 1 buyer.

[b] They consider two suppliers' structures: symmetric and asymmetric cost and yield.

[c] False failure return.

[d] Information (belief).

[e] Information.

[f] Retailer type.

in their coordination models (Cachon & Lariviere, 2005; Giannoccaro & Pontrandolfo, 2004; Giri & Sarker, 2019; Lan et al., 2018; Liu et al., 2017; Nie & Du, 2017; Sarkar & Bhala, 2021; Segal & Whinston, 2002; Xu et al., 2023).

Our literature review reveals that dual-sourcing supply chain coordination remains largely unstudied, despite the strategic importance of dual- or multi-sourcing in achieving supply chain resilience. Despite several contract types exhibiting effective supply chain coordination, research on the coordination of dual-sourcing supply chains under the TBS policy is notably scarce. Therefore, this study focuses on coordinating a dual-sourcing supply chain network under the TBS policy. Accordingly, we establish the necessary conditions for feasible coordination and show the structure of the contract types that make coordination possible, with the quantity discount contract as a particular example. In the next section, we describe the exact problem setting in more detail.

3. Problem description

We consider a two-echelon supply chain consisting of a *buyer* (denoted as B) in the first echelon and two suppliers (for the same item) in the second echelon. The first supplier, who we name the *regular supplier* (denoted as r), offers the item at a lower wholesale price (w_r), but has a longer lead time (l_r). The second supplier, who we name the *expedited supplier* (denoted as e) has a shorter leadtime ($l_e < l_r$), but charges a higher wholesale price ($w_e > w_r$) for the same item. In each period t, the buyer faces a stochastic, independent, and identically distributed demand (i.i.d) (D_t) with an expected demand of μ. Unmet demand is backordered and penalized with a unit backorder cost b and any inventory is charged a unit inventory holding cost h.

The buyer employs the TBS policy to manage its inventory, which involves ordering every period t the fixed quantity (Q_r) from the regular supplier and ordering a variable quantity (Q_e^t) from the expedited supplier to handle the demand uncertainty. More precisely, the buyer applies a basestock policy to decide on Q_e^t. See Section 3.1 for more details on the TBS policy. The basestock policy is the optimal policy in such a setting (Janakiraman et al., 2015). The suppliers' production or order decision-making is not considered in this study, and therefore, we assume that only the buyer is managing his inventories. Consequently, only the buyer is charged inventory holding and backorder costs.

We investigate two scenarios for coordinating the supply chain. In the first scenario, the expedited supplier is considered a spot market, and the buyer and the regular supplier are in a noncooperative game. This situation is referred to as a *two-player game*. In the second scenario, the expedited supplier is considered a player within the supply chain, resulting in a *three-player game*. For both scenarios, we study the game dynamics and whether coordination can be achieved to enhance the supply chain performance. We refer to the first scenario as the *single-coordination* case and in the second scenario as the *double-coordination* case. We use the following notations for the model formulation and analyses in this paper.

Π_B, Π_r, and Π_e	Profit of the buyer, regular supplier, and expedited supplier, respectively
Π_{Br}	Total expected profit of the buyer and the regular supplier
Π	Total expected profit of the buyer and both suppliers
p	Unit selling price by the buyer to its customers
w_r and w_e	Unit wholesale price of the regular and expedited suppliers, respectively
Δw	Difference in the unit wholesale prices ($\Delta w = w_e - w_r$)
Q_r and Q_e	Quantity ordered from the regular and expedited suppliers, respectively
c_r and c_e	Unit purchasing (or manufacturing) cost for the regular and expedited suppliers, respectively
h and b	Unit inventory holding and backorder costs for the buyer
$\bar{w}_e$	Maximum allowed price for the expedited supplier
S_{Q_r}	Basestock level for the expedited supplier

The following notations (Q_r, Q_e, Π_B, Π_r, Π_e, Π_{Br}, w_r and w_e) may include a superscript g, gg, c, or cc to indicate whether the notation corresponds to a two-players game, three-players games, single-coordination, or double-coordination, respectively.

3.1. TBS policy

We study a single-product multi-period inventory management system under stochastic demand. The buyer faces nonnegative stochastic demand from external customers. In this paper, we assume that only the buyer manages his inventory, whereas the suppliers apply a make-to-order system, i.e., their order or production decisions are made after receiving order decisions from the buyer.

The buyer replenishes its inventory by ordering from a regular r and expedited e supplier with the cost and leadtime structure mentioned in Section 3. The regular supplier procures or manufactures a unit at cost c_r and sells it to the buyer at price w_r, while the expedited supplier does so at cost c_e and sells at w_e, requiring that $w_e > w_r$. If the buyer cannot satisfy the demand from stock, unmet demand is backordered (or considered as lost sales in the final period of the horizon). The buyer adopts the TBS policy for inventory control, ordering a fixed and constant quantity, Q_r, from the regular supplier and varying amounts from the expedited supplier to cope with demand uncertainty.

The quantity ordered from the expedited supplier is determined by the basestock level S_{Q_r} which depends on Q_r. An order is placed at the expedited supplier whenever the inventory position IP^e (= net inventory plus all outstanding orders from both suppliers minus the backorder quantities is below S_{Q_r}. The expedited quantity Q_e^t is the difference needed to reach S_{Q_r}, i.e, $Q_e^t = \max(0, S_{Q_r} - \mathrm{IP}_t^e)$. It may happen that IP_t^e exceeds S_{Q_r}, a situation referred to as overshoot ($O(Q_r)$), which we will discuss towards the end of this section.

The basestock level S_{Q_r} is optimized by maximizing the buyer's profit function. The order of events under the TBS policy is as follows.

(a) The buyer evaluates its inventory position, which includes the initial stock and all outstanding orders from both suppliers:

$$\mathrm{IP}_t^e = x_t + \sum_{i=t-l_e}^{t-1} Q_e^i + \sum_{i=t-l_r}^{t-(l_r-l_e)} Q_r = x_t + \sum_{i=t-l_e}^{t-1} Q_e^i + l_e Q_r$$

(b) Given the inventory position, the buyer determines the quantity to order from the emergency supplier to reach the basestock level:

$$Q_e^t = (S_{Q_r} - \mathrm{IP}_t^e)^+$$

where $(\cdot)^+$ means $max\{\cdot, 0\}$.

(c) The buyer observes the actual demand D_t and updates the inventory level by adding the received quantities from both suppliers $y_t = x_t + Q_e^{t-l_e} + Q_r$ and then fulfills the demand (if possible).

(d) The inventory for the upcoming period x_{t+1} is set as the difference between the current inventory level y_t and the actual demand D_t, where a negative value indicates a shortage, which is backordered.

$$x_{t+1} = y_t - D_t$$

(e) The buyer's profit in period t becomes:

$$\Pi_B^t = p\min\{D_t, y_t\} - w_e Q_e^t - w_r Q_r - h(y_t - D_t)^+ - b(D_t - y_t)^+.$$

(f) The regular and expedited suppliers' profits are $\Pi_r^t = (w_r - c_r)Q_r$ and $\Pi_e^t = (w_e - c_e)Q_e^t$ respectively.

The overshoot distribution $O(Q_r)$, only dependent on Q_r, is obtained upon noting that the overshoot satisfies the following Lindley recursion relation $O_{t+1}(Q_r) = \max\{O_t(Q_r) - D_t + Q_r, 0\}$ and admits a stationary distribution, as shown in Loynes (1962). This stationary distribution satisfies a Lindley integral equation. The distribution may be obtained by using the Wiener–Hopf method, or by solving integral equation numerically or by simulating the overshoot process. In our numerical studies we simulated the overshoot process for a long period $T = 1000$, using Monte-Carlo simulation with 10000 replicates.

4. Two-players' game and single coordination

In this section, we present and study two-players' game and single-coordination problems involving the buyer and the regular supplier. The buyer decides on the optimal allocation of quantities Q_e^t and Q_r between the expedited and regular suppliers. Meanwhile, the regular supplier decides on the optimal selling price w_r knowing that the buyer will decide on order quantities that maximize its profit. Finally, the expedited supplier offers a high spot market price w_e (assumed to be fixed and known for now), and its expected profit is affected by the buyer's and regular supplier's decisions.

4.1. Two-players' Stackelberg game

We start with the buyer's problem. Given the wholesale price w_r set by the regular supplier, the buyer, acting as a follower, will maximize its expected long run average profit:

$$\Pi_B(Q_e, Q_r|w_r) = p\mu - w_e E_{Q_e}\{Q_e\} - w_r Q_r - hE_I\{I^+\} - bE_I\{I^-\},$$

where h and b are the unit holding and backorder costs, I the net inventory level, defined as the amount of inventory on hand less the backorders, and $I^+ = \max\{I, 0\}$ and $I^- = \max\{-I, 0\}$ are the excess on-hand inventory and backorder quantity, respectively.

Following Janakiraman et al. (2015) and noting that for the stationary solution $E_{Q_e}\{Q_e\} + Q_r = \mu$, for the case of ordering a constant quantity (Q_r) from the regular supplier, the expected long run average profit can be seen as:

$$\Pi_B(Q_r, S_{Q_r}|w_r) = p\mu - w_e(\mu - Q_r) - w_r Q_r - hE_{O_\infty}\{L_1(S_{Q_r} + O_\infty(Q_r))\} - bE_{O_\infty}\{L_2(S_{Q_r} + O_\infty(Q_r))\},$$

where S_{Q_r} denotes the basestock level for the expedited supplier that depends on Q_r (the decision for the expedited supplier is an up-to-level), $L_1(y) = E_{D_{l_e}}\{(y - D_{l_e})^+\}$, $L_2(y) = E_{D_{l_e}}\{(D_{l_e} - y)^+\}$, l_e is the lead time from the expedited supplier to the buyer, $D_{l_e} = \sum_{i=1}^{\ell_e+1} D_i$ and $O_\infty(Q_r)$ is the overshoot associated with Q_r. To simplify notations, we let F_e denote the distribution of D_{l_e}. Then, we have $L_1(y) = \int_0^y (y - t)dF_e(t)$ and $L_2(y) = \int_y^\infty (t - y)dF_e(t)$. Easy computations show that $L_1'(y) = F_e(y) \geq 0$, $L_2'(y) = F_e(y) - 1 \leq 0$ and $L_2(y) = L_1(y) - y + (\ell_e + 1)\mu$.

We denote H_{Q_r} as the stationary distribution function of the overshoot $O_\infty(Q_r)$, the expected long-run average profit can be rewritten as:

$$\Pi_B(Q_r, S_{Q_r}|w_r) = p\mu - w_e(\mu - Q_r) - w_r Q_r - h\int_0^\infty L_1(S_{Q_r} + x)dH_{Q_r}(x) - b\int_0^\infty L_2(S_{Q_r} + x)dH_{Q_r}(x).$$

Taking the derivative with respect to S_{Q_r}, the optimal up-to-level $S^*_{Q_r}$ solves

$$\frac{\partial \Pi_B^g}{\partial S_{Q_r}} = -(h + b)E\{F_e(S_{Q_r} + O_\infty(Q_r))\} + b = 0.$$

If we denote $M_{Q_r}(S_{Q_r}) = E\{F_e(S_{Q_r} + O_\infty(Q_r))\} = \int_0^\infty F_e(S_{Q_r} + x)dH_{Q_r}(x)$, the optimal up-to-level $S^*_{Q_r}$ solves $M_{Q_r}(S^*_{Q_r}) = b/(b + h)$. Replacing $S^*_{Q_r}$ in the profit expression yields

$$\Pi_B(Q_r|w_r) = (p - w_e)\mu + \Delta w Q_r - h\int_0^\infty L_1(S^*_{Q_r} + x)dH_{Q_r}(x) - b\int_0^\infty L_2(S^*_{Q_r} + x)dH_{Q_r}(x), \quad (1)$$

where $\Delta w = w_e - w_r$. Integration by parts shows that

$$\Pi_B(Q_r|w_r) = (p - w_e)\mu + \Delta w Q_r - hL_1(S^*_{Q_r}) - bL_2(S^*_{Q_r}) - \int_0^\infty [(h + b)F_e(S^*_{Q_r} + x) - b][1 - H_{Q_r}(x)]dx. \quad (2)$$

From Janakiraman et al. (2015), see also Appendix C.1 for a sketch of the proof, one can deduce that the function Π_B is concave in Q_r. Hence, the optimal regular order $Q_r^*(w_r)$ solves

$$\Delta w + \int_0^\infty [(h + b)F_e(S_{Q_r^*(w_r)^*} + x) - b]\frac{\partial H_{Q_r^*(w_r)}(x)}{\partial Q_r}dx = 0.$$

Remark 1. Using the recursive relation satisfied by the overshoot, namely $O_{t+1} = \max(0, O_t + Q_r - D_t)$, one sees that as Q_r increases, the overshoot also increases, implying that $H_{Q_r}(x)$ is decreasing in Q_r.

Anticipating the buyer's reaction, the regular supplier decides on the price w_r that maximizes its expected profit:

$$\Pi_r(w_r) = (w_r - c_r)Q_r^*(w_r). \quad (3)$$

For a given w_r, let $Q_r^*(w_r)$ be the optimal solution of Problem (2). Then, (w_r^g, Q_r^g) is a Stackelberg equilibrium solution if w_r^g solves Problem (3) and $Q_r^g \equiv Q_r^*(w_r^g)$. The existence of this equilibrium has been proven in Proposition 1.

For the expedited supplier, the expected profit equals:

$$\Pi_e^g = (w_e - c_e)E_{Q_e^g}\{Q_e^g\} = (w_e - c_e)(\mu - Q_r^g). \quad (4)$$

4.2. Single coordination

To evaluate the single coordination case, we first consider the integrated channel with one central decision maker who controls both the buyer and the regular supplier. The central authority decides on the optimal quantity Q_r^c that maximizes the total expected profit:

$$\Pi_{Br}(Q_r) = (p - w_e)\mu + (w_e - c_r)Q_r - hL_1(S^*_{Q_r}) - bL_2(S^*_{Q_r}) - \int_0^\infty [(h + b)F_e(S^*_{Q_r} + x) - b][1 - H_{Q_r}(x)]dx. \quad (5)$$

The optimal regular order Q_r^c solves

$$w_e - c_r + \int_0^\infty [(h + b)F_e(S^*_{Q_r^c} + x) - b]\frac{\partial H_{Q_r^c}(x)}{\partial Q_r}dx = 0. \quad (6)$$

Using the fact that $\frac{\partial^2 \Pi_B^g}{\partial w_r^g \partial Q_r^g} = -1 + \frac{\partial^2 \Pi_B^g}{\partial^2 Q_r^g}\frac{\partial Q_r^g}{\partial w_r^g} = 0$ at the optimum, we deduce that $\frac{\partial Q_r^g}{\partial w_r^g} \leq 0$. Therefore, replacing w_r in (2) by c_r in (5) will lead to an increase in Q_r. That is, $Q_r^c > Q_r^g$ which implies that

$$\Pi_B^c(Q_r^c|w_r^g) = (p - w_e)\mu + (w_e - w_r^g)Q_r^c - hL_1(S^*_{Q_r^c}) - bL_2(S^*_{Q_r^c}) - \int_0^\infty [(h + b)F_e(S^*_{Q_r^c} + x) - b][1 - H_{Q_r^c}(x)]dx < \Pi_B^g(Q_r^g|w_r^g), \quad (7)$$

$$\Pi_r^c(w_r^g) = (w_r^g - c_r)Q_r^c > (w_r^g - c_r)Q_r^g = \Pi_r^g(w_r^g). \quad (8)$$

Note that under coordination, the buyer's profit decreases and the regular supplier's profit increases. Moreover, the total supply chain profit (under coordination) Π_{Br} is greater than it would be without coordination (see Proposition 2). For coordination to occur, a contract must be developed between the buyer and the regular supplier that provides the buyer with an incentive to order the higher quantity Q_r^c that results in achieving coordination. To this end, w_r is assumed to be a function of the order quantity Q_r, that is $w_r = w_r(Q_r)$. The contract that achieves coordination is the one for which the buyer's optimal choice coincides with Q_r^c, that is $\frac{\partial \Pi_B^g}{\partial Q_r}|_{Q_r = Q_r^c} = 0$. This means that Q_r^c satisfies

$$w_e - w_r(Q_r^c) - Q_r^c\frac{\partial w_r(Q_r)}{\partial Q_r}|_{Q_r = Q_r^c} + \int_0^\infty [(h + b)F_e(S^*_{Q_r^c} + x) - b]\frac{\partial H_{Q_r}(x)}{\partial Q_r}|_{Q_r = Q_r^c}dx = 0.$$

Using Eq. (6), the above reduces to

$$c_r - w_r(Q_r^c) - Q_r^c\frac{\partial w_r(Q_r)}{\partial Q_r}|_{Q_r = Q_r^c} = 0. \quad (9)$$

Any contract $w_r(Q_r)$ satisfying (9) achieves coordination. For example, consider the following quantity discount contract:

$$w_r^c(Q_r) = \begin{cases} c_r + \frac{K_r^c}{Q_r^c} & \text{for } Q_r \le Q_r^c \\ c_r + \frac{K_r^c + L_r^c(\ln(Q_r/Q_r^c))^2}{Q_r} & \text{for } Q_r > Q_r^c, \end{cases}$$

for $K_r^c > 0$ and $L_r^c \le K_r^c$. This contract satisfies (9) and achieves coordination. $w_r^c(Q_r)$ is a decreasing function in Q_r since $\frac{\partial w_r^c(Q_r)}{\partial Q_r} = 0$ for $Q_r \le Q_r^c$ and $\frac{\partial w_r^c(Q_r)}{\partial Q_r} = -\frac{L_r^c(ln(Q_r/Q_r^c)-1)^2 + K_r^c - L_r^c}{Q_r^2} < 0$ for $Q_r > Q_r^c$. Hence $w_r^c(Q_r)$ represents a quantity-discount scheme. In addition, note that $\Pi_r = (w_r - c_r)Q_r = K_r^c + L_r^c(\ln(Q_r/Q_r^c))^2$ is increasing in Q_r.

The choice of K_r^c is an interesting problem, as it affects the profit going to the regular supplier. Under this type of quantity-discount contract, one can verify that the regular supplier's profit is given by $\Pi_r^c(w_r^c) = K_r^c$ and that the buyer's profit is $\Pi_B^c(Q_r^c|w_r^c) = \Pi_{Br}^c(Q_r^c) - K_r^c$. Therefore, if $K_r^c \ge \Pi_r^g$ is chosen, the regular supplier will find coordination beneficial. As long as K_r^c is smaller than $\Pi_{Br}^c - \Pi_B^g$, the buyer will also find coordination beneficial. Therefore, to convince both parties to coordinate, K_r^c should be chosen between Π_r^g and $\Pi_{Br}^c - \Pi_B^g$. This interval is always nonempty because $\Pi_{Br}^c \ge \Pi_B^g + \Pi_r^g$. Therefore, the choice of K_r^c ($\Pi_r^g \le K_r^c \le \Pi_{Br}^c - \Pi_B^g$) depends on the buyer and regular supplier's bargaining powers. See Appendix A for a brief discussion on how K_r^c can be set.

5. Three-players' game and double coordination

In this section, we study the three-players' game and double coordination problems involving the buyer and the expedited and regular suppliers. We assume that the buyer decides on the optimal allocation of quantities Q_e^t and Q_r between the expedited and regular suppliers. Meanwhile, the regular and expedited suppliers simultaneously decide on their optimal selling prices w_r and w_e, knowing that the buyer will decide on order quantities that maximize its profit.

5.1. Double Stackelberg game

Given the wholesale prices w_r and w_e set by the regular and expedited suppliers, the buyer, acting as a follower, will maximize its expected profit:

$$\Pi_B(Q_r|w_r, w_e) = (p - w_e)\mu + (w_e - w_r)Q_r - hL_1(S_{Q_r}^*) - bL_2(S_{Q_r}^*) - \int_0^\infty [(h+b)F_e(S_{Q_r}^* + x) - b][1 - H_{Q_r}(x)]dx. \tag{10}$$

The optimal regular order $Q_r^*(w_r, w_e)$ solves

$$\Delta w + \int_0^\infty [(h+b)F_e(S_{Q_r^*(w_r,w_e)}^* + x) - b]\frac{\partial H_{Q_r^*(w_r,w_e)}(x)}{\partial Q_r}dx = 0.$$

Anticipating the buyer's reaction, the regular supplier decides on the price w_r to maximize its expected profit

$$\Pi_r(w_r) = (w_r - c_r)Q_r^*(w_r, w_e) \tag{11}$$

and the expedited supplier decides on the price w_e to maximize its expected profit:

$$\Pi_e(w_e) = (w_e - c_e)(\mu - Q_r^*(w_r, w_e)). \tag{12}$$

Problems (11) and (12) are solved simultaneously to reach a Nash equilibrium, that is, optimal supplier prices are determined where the supplier has no incentive to deviate from its set price if the other supplier's price is considered. For a given (w_r, w_e), let $Q_r^*(w_r, w_e)$ be the optimal solution of Problem (10). Then $(w_r^{gg}, w_e^{gg}, Q_r^{gg})$ is a Stackelberg equilibrium solution if (w_r^{gg}, w_e^{gg}) is a Nash equilibrium of Problems (11)–(12) and $Q_r^{gg} \equiv Q_r^*(w_r^{gg}, w_e^{gg})$. The solution of such an equilibrium exists, see the remark after the proof of Proposition 2 in Appendix C.2.

5.2. Double coordination

We first consider the integrated channel where one central decision maker controls the buyer and the two suppliers. The central authority will select the optimal quantity Q_r^{cc} that maximizes the total expected profit:

$$\Pi(Q_r) = (p - c_e)\mu + (c_e - c_r)Q_r - hL_1(S_{Q_r}^*) - bL_2(S_{Q_r}^*) - \int_0^\infty [(h+b)F_e(S_{Q_r}^* + x) - b][1 - H_{Q_r}(x)]dx. \tag{13}$$

The optimal regular order Q_r^{cc} solves

$$c_e - c_r + \int_0^\infty [(h+b)F_e(S_{Q_r^{cc}}^* + x) - b]\frac{\partial H_{Q_r^{cc}}(x)}{\partial Q_r}dx = 0. \tag{14}$$

Remark 2. If the expedited supplier's unit profit is greater than that of the regular supplier ($w_e^{gg} - w_r^{gg} > c_e - c_r$), then reducing $w_e - w_r$ from (10) to $c_e - c_r$ in (13) will lead to a decrease in Q_r. Therefore $Q_r^{cc} < Q_r^{gg}$, which implies that

$$\Pi_B^{cc}(Q_r^{cc}|w_r^{gg}, w_e^{gg}) = (p - w_e^{gg})\mu + (w_e^{gg} - w_r^{gg})Q_r^{cc} - hL_1(S_{Q_r^{cc}}^*) - bL_2(S_{Q_r^{cc}}^*) - \int_0^\infty [(h+b)F_e(S_{Q_r^{cc}}^* + x) - b][1 - H_{Q_r^{cc}}(x)]dx < \Pi_B^{gg}(Q_r^{gg}|w_r^{gg}, w_e^{gg}), \tag{15}$$

$$\Pi_r^{cc}(w_r^{gg}) = (w_r^{gg} - c_r)Q_r^{cc} < (w_r^{gg} - c_r)Q_r^{gg} = \Pi_r^{gg}(w_r^{gg}), \tag{16}$$

$$\Pi_e^{cc}(w_e^{gg}) = (w_e^{gg} - c_e)(\mu - Q_r^{cc}) > (w_e^{gg} - c_e)(\mu - Q_r^{gg}) = \Pi_e^{gg}(w_e^{gg}). \tag{17}$$

If the expedited supplier's unit profit is less than or equal to that of the regular supplier ($w_e^{gg} - w_r^{gg} \le c_e - c_r$), modifying $w_e - w_r$ from (10) to $c_e - c_r$ in (13) will lead to an increase in Q_r and, as a result, $Q_r^{cc} > Q_r^{gg}$. In this case, $\Pi_B^{cc}(Q_r^{cc}|w_r^{gg}, w_e^{gg}) < \Pi_B^{gg}(Q_r^{gg}|w_r^{gg}, w_e^{gg})$, $\Pi_r^{cc}(w_r^{gg}) \ge \Pi_r^{gg}(w_r^{gg})$ and $\Pi_e^{cc}(w_e^{gg}) < \Pi_e^{gg}(w_e^{gg})$.

The challenge is to design two contracts - one between the buyer and the expedited supplier and another between the buyer and the regular supplier - that allow coordination to be achieved. To this end, w_e and w_r are assumed to be functions of the order quantities $Q_e = \mu - Q_r$ and Q_r, respectively. That is, $w_e = w_e(\mu - Q_r)$ and $w_r = w_r(Q_r)$. The contracts that achieve coordination are the ones for which the maximum expected profit Π_B^{gg} coincides with the quantity Q_r^{cc}, that is $\frac{\partial \Pi_B^{gg}}{\partial Q_r}|_{Q_r = Q_r^{cc}} = 0$. This means that Q_r^{cc} satisfies:

$$w_e(\mu - Q_r^{cc}) - (\mu - Q_r^{cc})\frac{\partial w_e(\mu - Q_r)}{\partial Q_r}|_{Q_r = Q_r^{cc}} - w_r(Q_r^{cc}) - Q_r^{cc}\frac{\partial w_r(Q_r)}{\partial Q_r}|_{Q_r = Q_r^{cc}} + \int_0^\infty [(h+b)F_e(S_{Q_r^{cc}}^* + x) - b]\frac{\partial H_{Q_r}(x)}{\partial Q_r}|_{Q_r = Q_r^{cc}}dx = 0.$$

Using Eq. (14), the above reduces to:

$$c_r - c_e + w_e(\mu - Q_r^{cc}) - (\mu - Q_r^{cc})\frac{\partial w_e(\mu - Q_r)}{\partial Q_r}|_{Q_r = Q_r^{cc}} - w_r(Q_r^{cc}) - Q_r^{cc}\frac{\partial w_r(Q_r)}{\partial Q_r}|_{Q_r = Q_r^{cc}} = 0. \tag{18}$$

Eq. (18) will hold true if:

$$-c_e + w_e(\mu - Q_r^{cc}) - (\mu - Q_r^{cc})\frac{\partial w_e(\mu - Q_r)}{\partial Q_r}|_{Q_r = Q_r^{cc}} = 0, \tag{19}$$

$$c_r - w_r(Q_r^{cc}) - Q_r^{cc}\frac{\partial w_r(Q_r)}{\partial Q_r}|_{Q_r = Q_r^{cc}} = 0. \tag{20}$$

Any contracts $w_e(\mu - Q_r)$ and $w_r(Q_r)$ that satisfy (19) and (20) will result in coordination. For example, consider the following quantity-discount contracts:

$$w_e^{cc}(\mu - Q_r) = \begin{cases} c_e + \frac{K_e^{cc}}{\mu - Q_r^{cc}} & \text{for } \mu - Q_r \le \mu - Q_r^{cc} \\ c_e + \frac{K_e^{cc} + L_e^{cc}(\ln((\mu - Q_r)/(\mu - Q_r^{cc})))^2}{\mu - Q_r} & \text{for } \mu - Q_r > \mu - Q_r^{cc}, \end{cases}$$

$$w_r^{cc}(Q_r) = \begin{cases} c_r + \frac{K_r^{cc}}{Q_r^{cc}} & \text{for } Q_r \le Q_r^{cc} \\ c_r + \frac{K_r^{cc} + L_r^{cc}(\ln(Q_r/Q_r^{cc}))^2}{Q_r} & \text{for } Q_r > Q_r^{cc}, \end{cases}$$

$$c_e + \frac{K_e^{cc}}{\mu - Q_r^{cc}} > c_r + \frac{K_r^{cc}}{Q_r^{cc}},$$

for $K_e^{cc} > 0$, $L_e^{cc} < K_e^{cc}$, $K_r^{cc} > 0$, and $L_r^{cc} < K_r^{cc}$. These two contracts satisfy (19) and (20) and achieve coordination. $w_e^{cc}(\mu - Q_r)$ and $w_r^{cc}(Q_r)$ are decreasing functions in $\mu - Q_r$ and Q_r, respectively. Hence, $w_e^{cc}(\mu - Q_r)$ and $w_r^{cc}(Q_r)$ represent quantity discount schemes. The condition $c_e + \frac{K_e^{cc}}{\mu - Q_r^{cc}} > c_r + \frac{K_r^{cc}}{Q_r^{cc}}$ ensures that $w_e^{cc}(\mu - Q_r^{cc}) > w_r^{cc}(Q_r^{cc})$.

The values of K_e^{cc} and K_r^{cc} are crucial to determine the feasibility of achieving supply chain coordination and profit distribution in the supply chain. Under the above quantity-discount contracts, the expedited supplier's profit is given by $\Pi_e^{cc}(w_e^{cc}) = K_e^{cc}$, the regular supplier's profit by $\Pi_r^{cc}(w_r^{cc}) = K_r^{cc}$, and the buyer's profit by $\Pi_B^{cc}(Q_r^{cc}|w_r^{cc}, w_e^{cc}) = \Pi^{cc} - K_e^{cc} - K_r^{cc}$. Therefore, choosing $K_e^{cc} \geq \Pi_e^{gg}$ and $K_r^{cc} \geq \Pi_r^{gg}$ will cause the expedited and regular suppliers to find coordination beneficial. As long as $K_e^{cc} + K_r^{cc}$ is smaller than $\Pi^{cc} - \Pi_B^{gg}$, the buyer will also find coordination beneficial. Therefore, to convince all three parties to coordinate, values of K_e^{cc} and K_r^{cc} should be chosen such that $K_e^{cc} \geq \Pi_e^{gg}$, $K_r^{cc} \geq \Pi_r^{gg}$ and $K_e^{cc} + K_r^{cc} < \Pi^{cc} - \Pi_B^{gg}$. Note that this interval is always nonempty because $\Pi^{cc} \geq \Pi_B^{gg} + \Pi_r^{gg} + \Pi_e^{gg}$. Moreover, we need to ensure that $c_e - c_r > \frac{K_r^{cc}}{Q_r^{cc}} - \frac{K_e^{cc}}{\mu - Q_r^{cc}}$ to satisfy the constraint $w_e^{cc}(\mu - Q_r^{cc}) > w_r^{cc}(Q_r^{cc})$. The values of K_e^{cc} and K_r^{cc} depend on the buyer's and suppliers' bargaining powers. See Appendix B for a brief discussion on how these parameters can be set.

6. Analysis and results

In Sections 4 and 5, we presented the formulation, analysis, and results of the two- and three-player games as well as for single and double coordination. In this section, we present detailed analytical results and numerical experiments that show the effect of several model parameters on the games and coordination. Section 6.1 provides all analytical results for the games and coordination. Section 6.2 presents the key dynamics of the games from the perspective of each of the three players, which would help the reader better interpret our results. All numerical experiments are provided in Sections 6.3 and 6.4. To inform the reader about the main results, the following values were chosen as the default parameter values: $p = 15$, $h = 1$, $b = 5$, $c_r = 1$, $c_e = 4$, $\mu = 10$, $l_e = 0$, $l_r = 1$ and $w_e = 7$. These values are proportionally realistic values and close to values chosen in the literature (Arts & Kiesmüller, 2013; Hekimoğlu & Scheller-Wolf, 2023). Furthermore, in our numerical experiments provided in Sections 6.3 and 6.4, we present the results of our numerical experiments in which several parameter values have been varied. We assume that any firm will cease its operation (go out of business) if the (long-run) expected profit is zero or negative, as commonly assumed in the literature (Cao et al., 2024). Note that we continue assuming the contracts introduced in Sections 4.2 and 5.2 to achieve coordination.

6.1. Analytical results

In this section, we present the analytical results that include the effect of several model parameters on the games and coordination. All proofs of these results are presented in Appendix C.

Under the two-players' game (between the buyer and regular supplier), the regular supplier's optimal price w_r^g is between its unit purchase (or manufacturing) cost c_r and the expedited supplier's price w_e. In Section 6.2, we will observe that w_r^g is just below w_e. On the other hand, the emergency supplier sets its price w_e^g either equal to w_r, which results in the regular supplier going out of business, or the expedited supplier will charge the maximum possible price $\bar{w}_e$. These price settings of the suppliers is presented in Proposition 1.

Proposition 1. *For fixed w_e, $c_r < w_r^g < w_e$; For fixed w_r, $w_e^g = w_r$ or $w_e^g = \bar{w}_e$.*

Single coordination makes the buyer and regular supplier secure at least (and most likely more) profit than if the two parties do not coordinate, as presented in Proposition 2. Hence, the two parties have an incentive to coordinate, as the sum of their profits increases. Consequently, the expedited supplier's profit will decrease, as will be shown in Section 6.3.

Proposition 2. $\Pi_{Br}^c(Q_r^c) \geq \Pi_B^g(Q_r^g) + \Pi_r^g(Q_r^g)$.

We find that the larger the expedited supplier's (or spot market) price, the more the buyer will order from the regular supplier, i.e., the higher the fixed order quantity, which results in an increased profit for the regular supplier. On the other hand, when the expedited supplier's (or spot market) price increases, it negatively impacts the buyer's profit. This is presented in Proposition 3.

Proposition 3.

a. Q_r^c is increasing in w_e.
b. Π_r^g is increasing in w_e; Π_{Br}^c and $\Pi_{Br}^c - \Pi_r^g$ are decreasing in w_e.

As the regular supplier's unit purchase (or manufacturing) cost (c_r) increases, the regular supplier will have to charge the buyer a higher price, leading to a decreased profit for the regular supplier and the buyer (whether they coordinate or not). However, the expedited supplier is benefiting from an increased c_r. These results are presented in Proposition 4.

Proposition 4.

a. Q_r^c and Q_r^{cc} are decreasing in c_r.
b. Π_e^c is increasing in c_r; Π_r^g, Π_{Br}^c, $\Pi_{Br}^c - \Pi_r^g$ and Π^{cc} are decreasing in c_r.

Under double coordination, an increase of the expedited supplier's unit purchase (or manufacturing) price c_e leads to a larger allocation of orders towards the regular supplier. Nevertheless, the total supply chain profit decreases. This is presented in Proposition 5.

Proposition 5.

a. Q_r^{cc} is increasing in c_e.
b. Π^{cc} is decreasing in c_e.

An increase in the unit inventory holding cost h (and unit backorder cost b) results in ordering more from the expedited supplier to avoid expensive leftovers (shortages), which negative impacts the profits. Propositions 6 and 7 are presenting this.

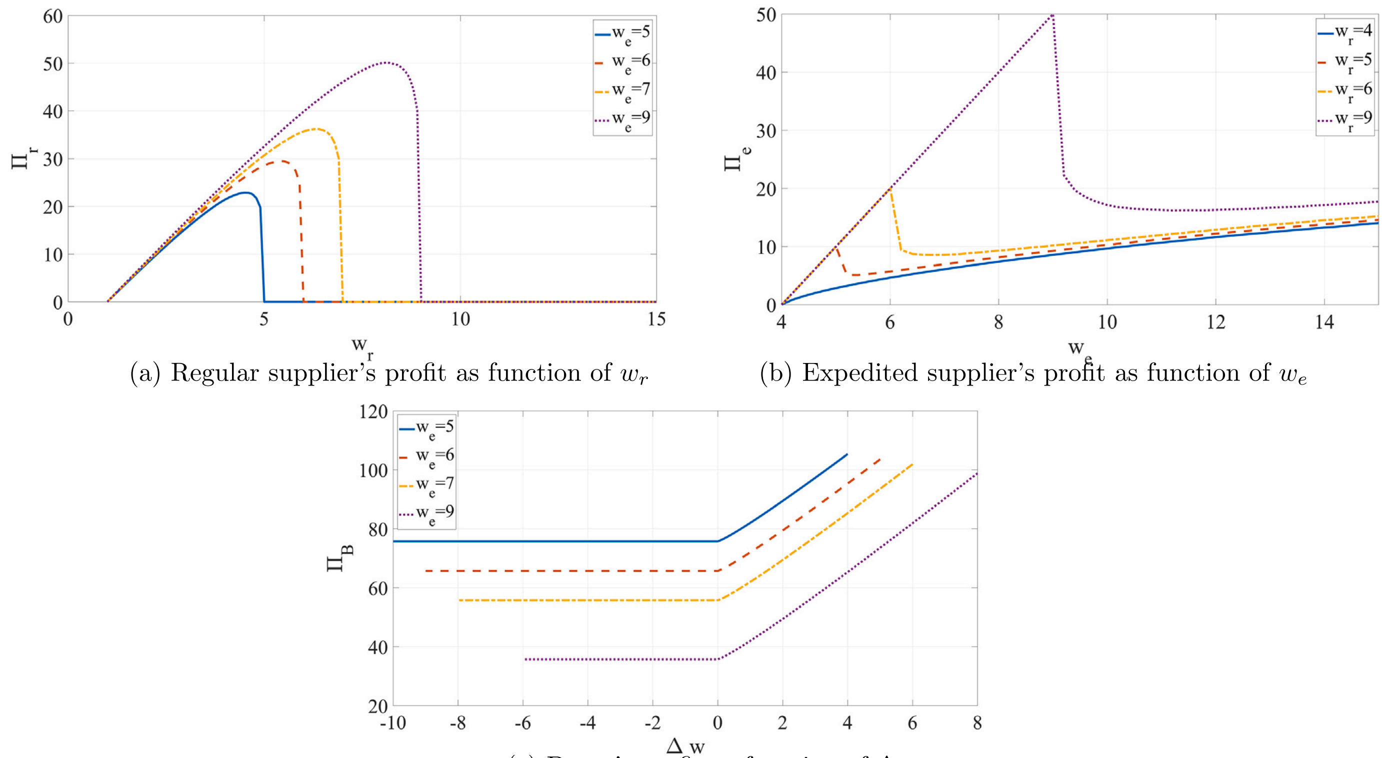
Proposition 6. *Π_{Br}^c and Π^{cc} are decreasing in h.*

Proposition 7. *Π_{Br}^c and Π^{cc} are decreasing in b.*

6.2. Game dynamics

The two- and three-players' games have interesting dynamics. First, if we focus on the two-players' game (between the buyer and the regular supplier), w_e becomes an exogenous parameter. If the regular supplier charges the buyer $w_r \geq w_e$, the regular supplier will be *out of business*, that is, $Q_r^g = 0$, because in that case, the buyer will only order from the expedited supplier. This is presented in Fig. 1(a), which shows Π_r^g as a function of w_r for various values of w_e. Indeed, the regular supplier's profit is only positive if $w_r < w_e$. Fig. 1(a) shows that the regular supplier's expected profit is maximized at a value *slightly* lower than w_e. In other words, the regular supplier will charge the buyer a w_r just below w_e, resulting in a small $\Delta w = w_e - w_r$.

Alternatively, if w_r becomes an exogenous parameter, the expedited supplier's strategy (optimal w_e) depends on w_r, as shown in Fig. 1(b).



(a) Regular supplier's profit as function of w_r

(b) Expedited supplier's profit as function of w_e

(c) Buyer's profit as function of Δw

Fig. 1. Visualization of the games' main dynamics for the supply chain parties.

More precisely, in case w_r is relatively high, such as when $w_r = 9$ in Fig. 1(b), the expedited supplier will opt to set w_e equal to w_r. This will maximize the expedited supplier's profit and result in the regular supplier going out of business, resulting in the expedited supplier becoming the only supplier for the buyer. Alternatively, if w_r is not high, such as when $w_r = 5$ in Fig. 1(b), the expedited supplier will charge the maximum possible w_e to maximize its profit. Despite the order quantity from the buyer to the expedited supplier decreasing with w_e, the increased unit profit makes it attractive for the expedited supplier to charge the maximum possible w_e.

The dynamics of the three-players' game are extensions of the two-players' game. In that case, the expedited supplier applies an aggressive strategy: either move to put the regular supplier out of business by setting w_e equal to w_r or set w_e equal to p, whichever maximizes the expedited supplier's expected profit. When the expedited supplier's pricing strategy threatens to put the regular supplier out of business, the regular supplier responds by lowering w_r as long as $w_r \geq c_r$. Given the expedited supplier's aggressive strategy, an equilibrium is achieved, wherein no supplier has an incentive to deviate from w_r^{gg} and w_e^{gg}, as illustrated in Figs. 1(a) and 1(b).

The buyer's profit decreases with w_e and w_r, but increases with Δw, see Fig. 1(c). Hence, there are conflicting interests between the two suppliers on the one hand and the buyer on the other hand. The suppliers generally maximize their profit by minimizing Δw whereas the buyer prefers to maximize Δw.

6.3. Numerical results of two-players' game and single coordination

In this section, we present numerical results of the two-players' game (between the buyer and the regular supplier) and the single-coordination case, as presented in Section 4. The expedited supplier is an external party and is considered as the spot market. In any case, the expedited supplier is not involved in the game; coordination occurs only between the buyer and the regular supplier.

6.3.1. Effect of the spot market price (w_e)

Despite the expedited supplier being an external party, its price-setting (w_e) affects the decisions of the buyer and the regular supplier and consequently affects coordination between them. Q_r^c is increasing in w_e, as proven in Proposition 3. The same holds for Q_r^g, which is also increasing in w_e, as shown in Fig. 2(a). That means the higher the spot market price w_e, the larger the regular orders. In addition, Fig. 2(a) confirms that $Q_r^c > Q_r^g$, which means that under coordination, the regular orders increase. Fig. 2(b) shows that the buyer's profit is between $\Pi_B^{c,min} = \Pi_B^g$ and $\Pi_B^{c,max} = \Pi_{Br}^c - \Pi_r^g$ and that, depending on K_r^c, coordination may improve the buyer's profit. As discussed in Section 4.2, K_r^c is a parameter that denotes the fraction of the total profit (due to coordination) allocated to the regular supplier, which is usually the result of power distribution within the supply chain. Note that K_r^c should stay within a nonempty interval (between Π_r^g and $\Pi_{Br}^c - \Pi_B^g$) to make coordination possible. Fig. 2(b) shows that the buyer's profit Π_B^g decreases in w_e. It also shows that $\Pi_B^{c,max} = \Pi_{Br}^c - \Pi_r^g$ decreases in w_e, as proven in Proposition 3. The opposite holds for the regular supplier's profit, which is increasing in w_e; see Fig. 2(b) and the proof in Proposition 3.

Fig. 2(c) shows that w_r^g is increasing in w_e, but that the difference ($\Delta w = w_e - w_r^g$) is limited. As discussed in Section 6.2, it is optimal for the regular supplier to set its price w_r^g just under the spot market price w_e. The same figure also shows that under coordination, the regular supplier will offer a price discount w_r^c (between $w_r^{c,min} = c_r + \frac{\Pi_r^g}{Q_r^c}$ and $w_r^{c,max} = c_r + \frac{\Pi^c - \Pi_B^g}{Q_r^c}$) that results in a larger order from the buyer, as illustrated in Fig. 2(a).

Fig. 2(d) shows that the expedited supplier's profit increases in w_e and decreases when the buyer and regular supplier coordinate ($\Pi_e^c < \Pi_e^g$). The latter result arises because, under single coordination, the buyer orders more from the regular supplier ($Q_r^c > Q_r^g$). Hence, the total profit of the buyer and regular supplier increases when they coordinate ($\Pi_{Br}^c > \Pi_B^g + \Pi_r^g$), but decreases as the spot market price w_e increases, as shown in Fig. 2(d) and proven in Proposition 3.

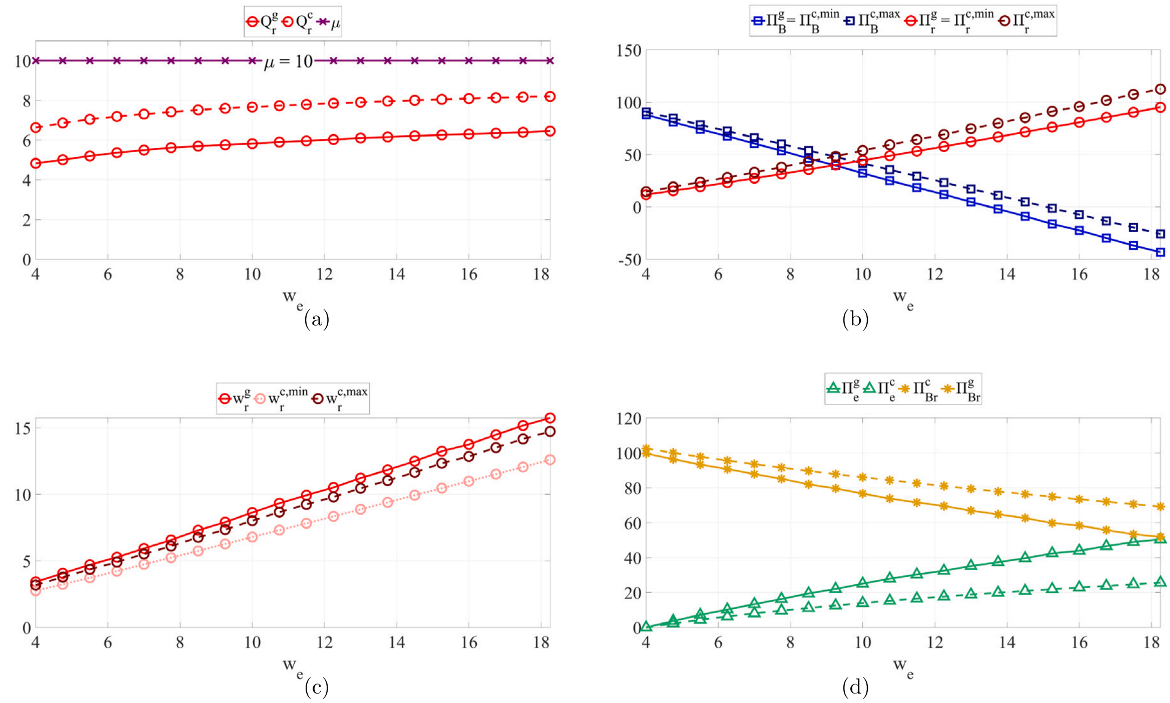


Fig. 2. Results of numerical experiments when varying w_e.

Note that in these experiments, we allowed w_e to exceed $p = 15$. In case $w_e > p$, Q_r keeps increasing in w_e, but still, $Q_r < \mu$. As a result, $E\{Q_e\}$ remains positive and Π_e keeps increasing (Fig. 2(d)). On the other hand, the buyer's expected profit becomes negative, see Fig. 2(b). In our numerical setting, this occurs when $w_e \simeq 13.5$. Consequently, the buyer will no longer be interested to operate in such an environment (where the long-run expected profit is negative), which makes him decide to go *out of business*.

6.3.2. Effect of the regular supplier's sourcing cost (c_r)

The regular supplier's sourcing cost (c_r) affects the profitability of the regular supplier and its pricing (w_r) decisions, and hence, coordination. Fig. 3(a) shows the impact of c_r on w_r with and without coordination. The regular supplier's pricing (w_r) increases with its sourcing cost (c_r), until $c_r = w_e = 7$. From this point, the regular supplier's sourcing cost c_r exceeds the spot market price w_e, meaning that the regular supplier is out of business, as can be seen in Fig. 3(b), resulting in a decreasing profit before $c_r = 7$ and zero profit starting from $c_r = 7$. Hence, the system turns into a single-sourcing system. Figs. 3(b) and 3(c) show that Q_r^c and Π_r^g are decreasing in c_r, as proven in Proposition 4. An increase in c_r negatively affects the profit of the buyer (see Fig. 3(c)) and the total supply chain (see Fig. 3(d) as well as the proof in Proposition 4). Meanwhile, the expedited supplier's profit increases in c_r, as illustrated in Fig. 3(d) and proven in Proposition 4, because larger order quantities will be allocated to the expedited supplier (see Fig. 3(b)).

6.3.3. Effects of the holding (h) and backorder (b) costs

Increasing the buyer's unit inventory holding cost h will cause the buyer to reduce the fixed order quantity from the regular supplier to avoid expensive leftovers. Instead, more orders will be placed with the expedited supplier, who reacts faster, to reduce the risk of higher inventory holding costs. Hence, the profit of the expedited supplier increases in h (see Fig. 4(a)), in opposition to the profits of the regular supplier and the buyer (see Figs. 4(a) and 4(b)), which decrease in h. The total supply chain profit decreases in h, as proven and shown in Proposition 6.

An increase in the backorder cost b negatively affects the buyer's profit - see Fig. 4(c) - but hardly affects supplier profit levels (Fig. 4(d)). This is because an increase in the buyer's backorder cost hardly affects ordering behavior towards the two suppliers. The total supply chain profit decreases in b, as proven in Proposition 7.

6.3.4. Effects of the lead time of the expedited supplier (l_e)

In the previous experiments, the lead time of the expedited supplier (l_e) was set equal to zero time periods. We present in this subsection the results of experiments in which l_e has been varied between 0 and 4. Fig. 5(a) shows that Q_r^g (as well as Q_r^c) increases in l_e. Also, w_r^g (as well as w_r^c) increases in l_e (Fig. 5(b)). As a result, the regular supplier's profit increases (Fig. 5(c)). On the other hand, an increase of l_e reduces Q_e, as the expedited supplier becomes less attractive for the buyer, resulting in a slight decrease of the expedited supplier's profit (Fig. 5(d)) as well as a decrease of the buyer's profit (Fig. 5(c)). The latter is due to a decreased responsiveness to meet the uncertain demand.

6.4. Numerical results of three-players' game and double coordination

In this section, we present numerical results for the three-players' game and double coordination, as presented in Section 5. The results of the experiments when varying h and b are not presented in this section, as they do not yield new insights compared to the results of the two-players' game and single coordination.

6.4.1. Effect of the expedited supplier's sourcing cost (c_e)

When the expedited supplier's sourcing cost c_e increases, the regular supplier reacts by increasing his equilibrium price w_r^{gg}. Hence, the buyer follows by decreasing Q_r^{gg}, which results in an increase of Q_e^{gg}. Therefore, an equilibrium is only maintained under an increased Q_e^{gg}. The optimal price the expedited supplier charges does not change, as it remains equal to the maximum allowable price. This effect can be seen in Fig. 6(a), which shows that Q_r^{gg} decreases in c_e. Simultaneously, the expedited supplier charges the buyer a fixed w_e^{gg} - see Fig. 6(b) - independent of c_e. Despite a fixed w_e^{gg} and an increased Q_e^{gg}, the expedited supplier's profit decreases - see Fig. 6(c) - which is due to

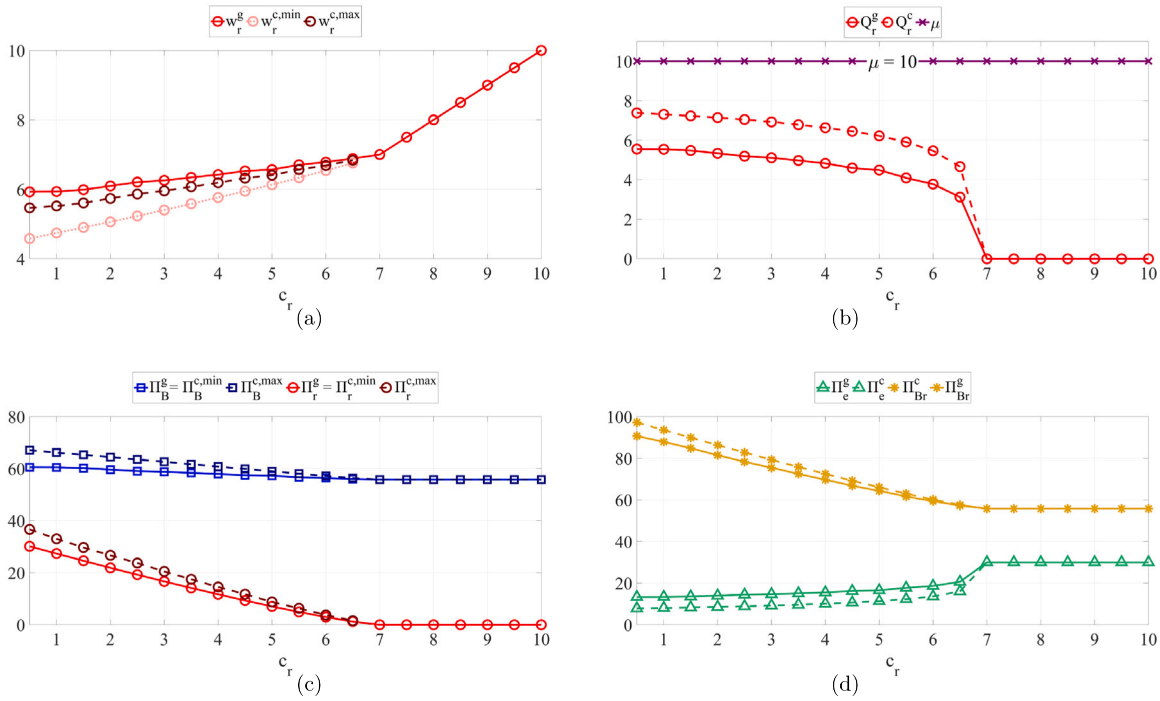


Fig. 3. Results of numerical experiments when varying c_r.

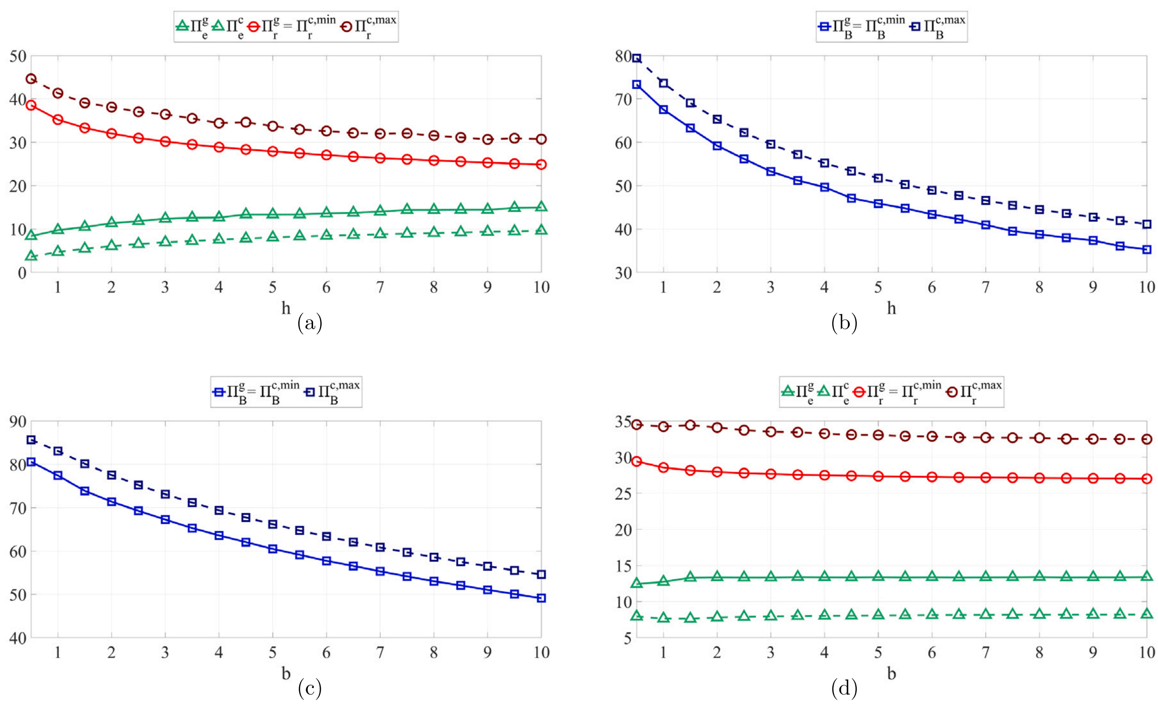


Fig. 4. Results of numerical experiments when varying h and b.

the increased c_e that reduces the expedited supplier's unit profit margin. Hence, the expedited supplier cannot fully compensate for the effect of increased c_e on its total profit.

However, the regular supplier clearly benefits from an increased c_e. Despite Q_r^{gg} decreasing in c_e, the regular supplier can charge a higher w_r^{gg}, because the expedited supplier is limited in reducing w_e^{gg} when c_e increases. Consequently, the regular supplier obtains greater latitude to charge a higher w_r^{gg}, which benefits the regular supplier - see Fig. 6(c).

An increase in c_e negatively affects the buyer's profit - see Fig. 6(d) - because the buyer is sourcing more from the (more expensive) expedited supplier. Despite the regular supplier's profit increasing in c_e, the buyer's and expedited supplier's profits decrease in c_e; therefore, the total supply chain profit also decreases in c_e, as proven in Proposition 5.

Unlike the game, under double coordination, Q_r^{cc} increases in c_e, as shown in Fig. 6(a). This is because, under double coordination, the system converts to one with a single decision authority. In that case,

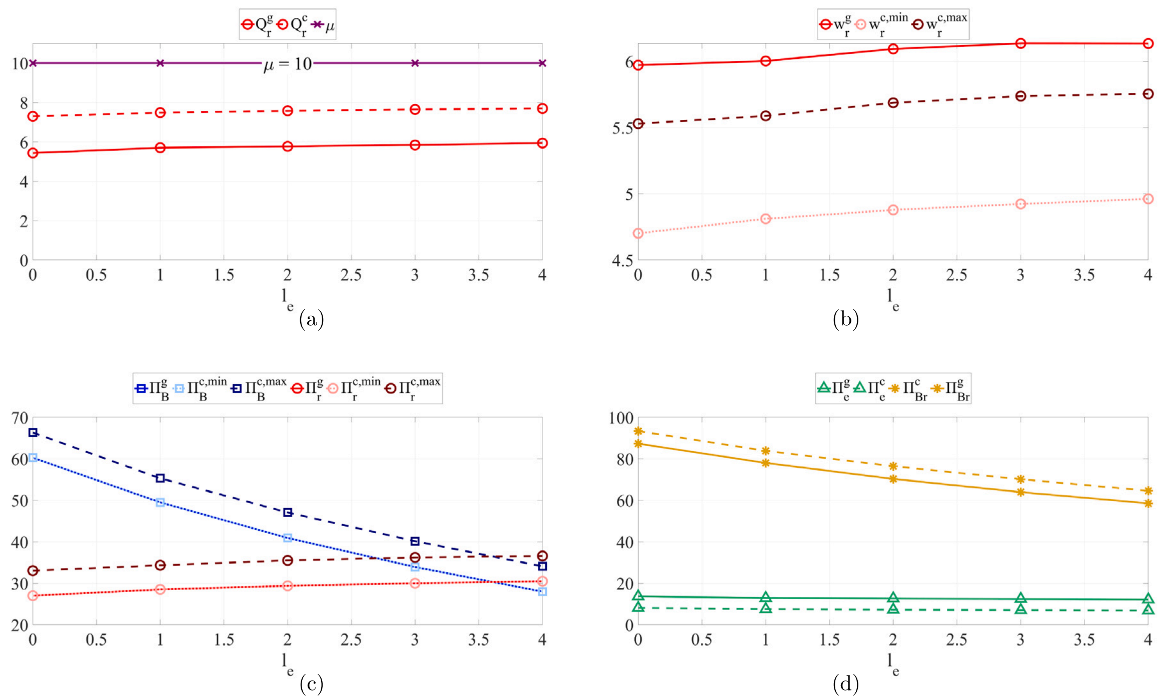


Fig. 5. Results of numerical experiments when varying l_e.

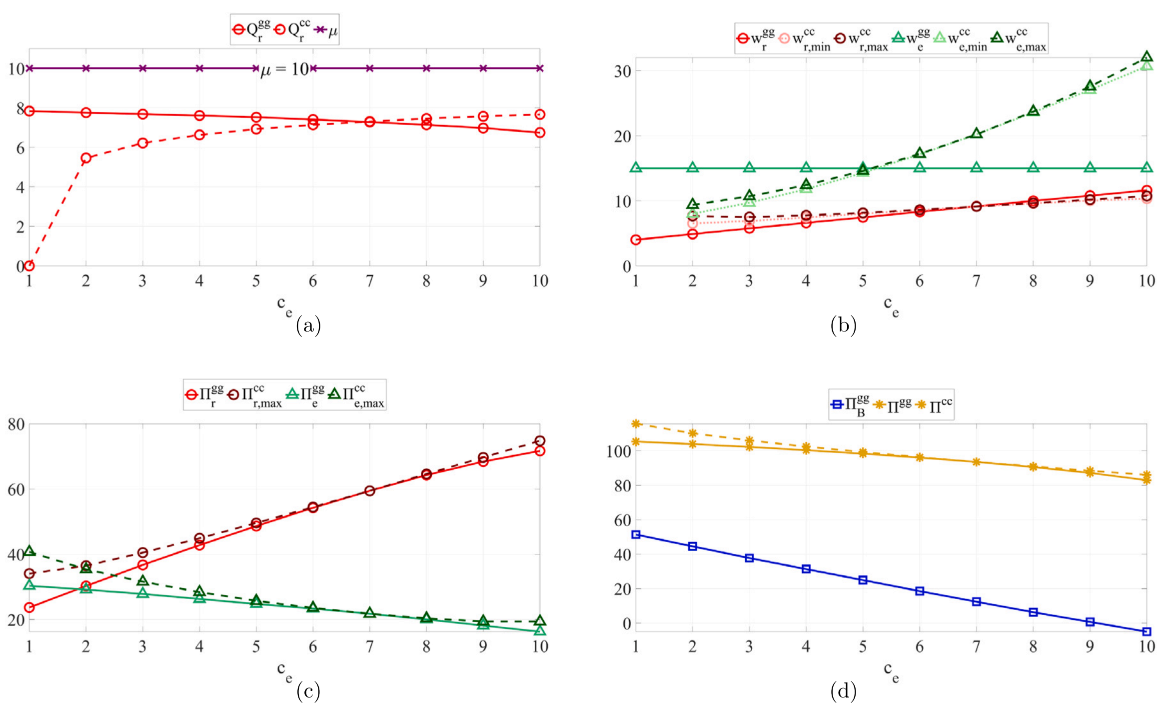


Fig. 6. Results of numerical experiments when varying c_e.

an increase in c_e leads to the allocation of a larger share of order quantities to the regular supplier, that is, Q_r^{cc} increases. We prove this in Proposition 5. As a result, Q_e^{cc} decreases in c_e, and therefore, the expedited supplier will charge a higher w_e^{cc} to make coordination possible, as shown in Fig. 6(b). Note that coordination is not possible when $c_e \geq 5$, because then, $w_e^{cc} > p$, which is unacceptable for the buyer. The regular supplier's price setting under double coordination is similar to the three-players' game, that is, w_r^{cc} is increasing in c_e with a minor adjustment to w_r^{cc} compared to w_r^{gg}.

6.4.2. Effect of regular supplier's sourcing cost (c_r)

In our numerical example with the default parameter values, equilibrium is achieved when the regular supplier sets $w_r^{gg} \simeq 6.6$ - see Fig. 7(a). This is independent of c_r. A further increase in w_r^{gg} will give an incentive to the expedited supplier to set w_e^{gg} such that the regular supplier goes out of business, as discussed in Section 6.2. If c_r exceeds w_r^{gg}, Q_r^{gg} drops to zero (Fig. 7(b)), because the regular supplier prefers zero profit over a negative profit (Fig. 7(c)). Hence, when c_r grows large, the system turns into a single-sourcing system where

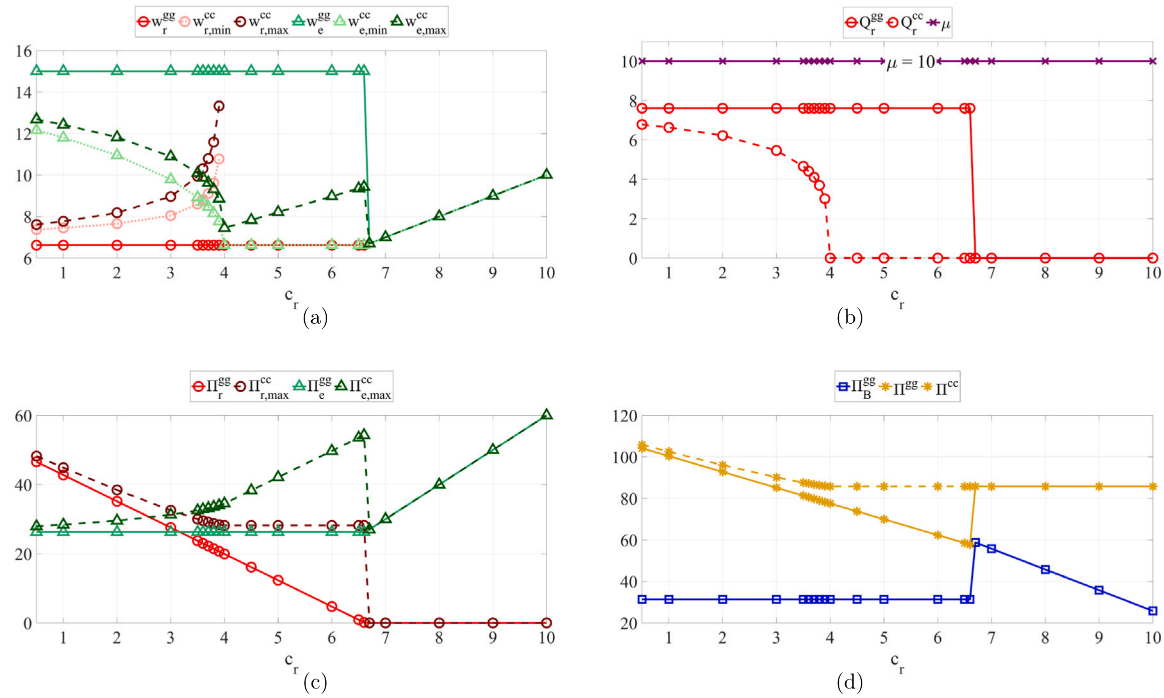


Fig. 7. Results of numerical experiments when varying c_r.

the expedited supplier replaces the regular supplier and charges the buyer w_e^{gg}, equal to c_r. The buyer benefits from such a single-sourcing system, as a lower w_e^{gg} and increased flexibility in order quantities increase the buyer's profit, as shown in Fig. 7(d). Although the buyer's profit increases when the regular supplier is put out of business, it declines as w_e^{gg} increases in c_r. In double coordination, as c_r increases, the regular supplier must increase w_r^{cc} to accept the coordination (see Fig. 7(a)), resulting in a decreasing Q_r^{cc} (see Fig. 7(a)). Meanwhile, w_e^{cc} is decreasing in c_r, with the result that Q_e^{cc} and the expedited supplier's profit are increasing in c_r - see Figs. 7(b) and 7(c). Note that coordination is no longer possible when $c_r > c_e$.

Table 2
Minimum and maximum efficiency levels with varying model parameters.

# of players	Parameter	Minimum efficiency	Maximum efficiency
2	w_e	0.832	0.967
	c_r	0.933	1
	c_e	0.939	0.939
	h	0.930	0.951
	b	0.933	0.955
	l_e	0.906	0.935
3	c_r	0.727	1
	c_e	0.910	1
	h	0.979	0.984
	b	0.977	0.987

6.5. Coordination efficiencies

Table 2 shows the efficiency of the coordination (Cachon, 2003), that is, the added value of coordination compared to the game. In the two-players' setting, coordination efficiency is defined as $\frac{\Pi_{Br}^{g}}{\Pi_{Br}^{c}}$. In the three-players setting, the efficiency becomes $\frac{\Pi^{gg}}{\Pi^{cc}}$. Hence, the lower the efficiency value, the higher the added value of coordination.

We run several numerical experiments while varying the model parameters; Table 2 shows the minimum and maximum values of the coordination efficiencies for the two- and three players' games and each varied model parameter. The results show that coordination typically leads to an average profit increase of about 6% over all our experiments.

7. Insights

Despite that the dual sourcing system has been studied extensively within the literature (see Section 2), this study is the first to explicitly consider each party in the system (the buyer and the two suppliers) to be an independent entity with an own objective function not aligned with others. Hence, interesting games arise, as we analyzed in Sections 4 and 5. Below, we discuss multiple insights gained from this study.

In a game between two parties (the buyer and regular supplier) with the expedited supplier an external party, equilibrium will always be achieved. We show analytically the parameter setting under which this occurs. The essence of the game is that the regular supplier will always set a price just below the (exogenous) expedited supplier's (or spot market) price. However, the two parties (buyer and regular supplier) have an incentive to coordinate - the sum of their profits will increase when they coordinate. We show the type of contracts that will make coordination possible. The additional profit due to coordination is allocated based on the bargaining power of the two parties; however, in all cases, they will be better off with coordination. The expedited supplier, also seen as the spot market, is harmed when the other two coordinate, as its profit decreases.

Moreover, we show that when the expedited supplier joins the game, equilibrium is still achieved. In this three-players game, the behavior of the regular supplier is the same as in the two-players' game, that is, it sets its price just below the expedited supplier's price. However, the expedited supplier acts more aggressively. Depending on the regular supplier's price, the expedited supplier will either set its price to put the regular supplier out of business or charge the maximum possible price from the buyer. We discuss these interesting game dynamics in more detail in Section 6.2. Our analysis shows the parameter setting for equilibrium to occur in the three-players' game, such that no player has an incentive to deviate from its optimal decision. In addition, coordination between the three parties, which we refer to as double coordination, is possible; we present the conditions under which this can occur. We further show the contract setting that makes coordination between the three parties possible.

Moreover, we show the impact of several model parameters on the game and coordination based on numerical results in Sections 6.3 and 6.4 with the support of the analytical results as presented in Section 6.1. For example, when the buyer and regular supplier coordinate, the regular order quantity increases compared with the game. As a result, the regular supplier charges a lower price w_r. The expedited supplier's profit decreases when the buyer and regular supplier coordinate; however, in all cases, the higher its price setting (or the spot market price) w_e, the higher the expedited supplier's profit, the larger the regular orders, and the lower the buyer's profit. Moreover, we showed that if the regular supplier's sourcing cost increases, the regular order decreases, resulting in the buyer being charged a higher price w_r, but only if the unit profit is positive. Otherwise, the regular supplier will prefer to go out of business. An increase in the regular supplier's sourcing cost leads to increased profit for the expedited supplier, with the others' profits decreasing. An increase in the buyer's inventory holding cost leads to a reduction in the (fixed) regular order, and an increase in the expedited supplier's profit. An increase in the backorder cost negatively affects the buyer's profit.

8. Conclusions

In this paper, we investigate a two-echelon dual sourcing supply chain with a buyer and two suppliers (a regular and an expedited supplier) under the TBS policy. Assuming that each firm in this chain optimizes its individual objective function, resulting in interesting game dynamics, the possibility of coordination has been explored in this paper.

First, we analyze a game setting between the buyer and the regular supplier, assuming that the expedited supplier is not part of the game, which can be seen as a spot market. Based on our analysis, we find that the regular supplier will charge the buyer a price just below the expedited supplier's (or spot market's) price, resulting in a minimal difference between the suppliers' wholesale prices. However, the buyer benefits from an increased difference in the suppliers' prices. Our findings indicate that coordination between the buyer and the regular supplier enhances their combined profit but negatively affects the expedited supplier's profit. We derive the exact conditions allowing for coordination and show the minimum and maximum profit that each party in the supply chain can secure under coordination.

Second, we extended the game setting by including the expedited supplier as a third player and studied the double coordination scenario. In the case of a three-player's game, the regular supplier's pricing strategy does not change. However, the expedited supplier adopts a more aggressive strategy, namely, by attempting to put the regular supplier out of business or charging the maximum possible price.

Moreover, we prove and show numerically the impact of several model parameters under the two- and three-players' games and (single and double) coordination, revealing several interesting insights. For example, in the single coordination setting where the expedited supplier is a spot market, we demonstrate that the regular order quantity increases as the wholesale price of the expedited supplier increases; however, the buyer's profit decreases under coordination.

The insights from this study suggest the following topics for future research. The modeling approach can be extended by investigating the effects of varying lead times on the game dynamics and coordination. A shortened lead time requires investment or faster transportation modes, whereas a slower transportation mode extends the lead time. An interesting trade-off arises here that is worthy of future study. Another interesting extension of our work could involve considering the suppliers' inventory management. In such a case, the dynamics could gain an additional dimension, especially since the regular supplier does not face demand uncertainty, unlike the expedited supplier. This may impact the suppliers' price setting, as the expedited supplier may charge a higher price for placing safety stocks to hedge against demand uncertainty.

Acknowledgments

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Appendix A. Setting K_r^c

The choice of K_r^c ($\Pi_r^g \leq K_r^c \leq \Pi^c - \Pi_B^g$) depends on the buyer and regular supplier's bargaining powers. A few cases of how to set K_r^c would be:

- When $K_r^c = \Pi_r^g$, $\Pi_r^c = \Pi_r^{c,min} = \Pi_r^g$ and $\Pi_B^c = \Pi_B^{c,max} = \Pi^c - \Pi_r^g$, and the extra profit generated due to coordination will all go to the buyer.
- When $K_r^c = \Pi^c - \Pi_B^g$, $\Pi_r^c = \Pi_r^{c,max} = \Pi^c - \Pi_B^g$ and $\Pi_B^c = \Pi_B^{c,min} = \Pi_B^g$, and the extra profit generated due to coordination will all go to the regular supplier.
- When $K_r^c = \frac{\Pi_r^g + \Pi^c - \Pi_B^g}{2}$, $\Pi_B^c = \Pi_B^g + \frac{\Pi^c - \Pi_B^g - \Pi_r^g}{2}$ and $\Pi_r^c = \Pi_r^g + \frac{\Pi^c - \Pi_B^g - \Pi_r^g}{2}$, and the extra profit generated due to coordination will be equally split between the buyer and the regular supplier.

Alternatively, K_r^c can be set by formulating the problem as a general Nash-bargaining game problem (Nagarajan & Bassok, 2008; Palit & Brint, 2020).

Appendix B. Setting K_e^{cc} and K_r^{cc}

The values of K_e^{cc} and K_r^{cc} depend on the buyer's and the suppliers' bargaining powers. The following are a few cases of how these parameters could be set:

- When $K_e^{cc} = \Pi_e^{gg}$ and $K_r^{cc} = \Pi_r^{gg}$, $\Pi_e^{cc} = \Pi_e^{cc,min} = \Pi_e^{gg}$, $\Pi_r^{cc} = \Pi_r^{cc,min} = \Pi_r^{gg}$, $\Pi_B^{cc} = \Pi_B^{cc,max} = \Pi^{cc} - \Pi_e^{gg} - \Pi_r^{gg}$, and the extra profit generated from double coordination will all go the buyer.
- When $K_e^{cc} = \Pi_e^{gg}$ and $K_r^{cc} = \Pi^{cc} - \Pi_e^{gg} - \Pi_B^{gg}$, $\Pi_e^{cc} = \Pi_e^{cc,min} = \Pi_e^{gg}$, $\Pi_r^{cc} = \Pi_r^{cc,max} = \Pi^{cc} - \Pi_e^{gg} - \Pi_B^{gg}$, $\Pi_B^{cc} = \Pi_B^{cc,min} = \Pi_B^{gg}$, and the extra profit generated from double coordination will all go the regular supplier.
- When $K_e^{cc} = \Pi^c - \Pi_r^{gg} - \Pi_B^{gg}$ and $K_r^{cc} = \Pi_r^{gg}$, $\Pi_e^{cc} = \Pi_e^{cc,max} = \Pi^{cc} - \Pi_r^{gg} - \Pi_B^{gg}$, $\Pi_r^{cc} = \Pi_r^{cc,min} = \Pi_r^{gg}$, $\Pi_B^{cc} = \Pi_B^{cc,min} = \Pi_B^{gg}$, and the extra profit generated from double coordination will all go the expedited supplier.

Note that the condition $c_e - c_r > \frac{\Pi_r^{gg}}{Q_r^{cc}} - \frac{\Pi^{cc} - \Pi_r^{gg} - \Pi_B^{gg}}{\mu - Q_r^{cc}}$ is needed to ensure that $w_e^{cc,max}(\mu - Q_r^{cc}) > w_r^{cc,min}(Q_r^{cc})$, and hence allowing feasibility to achieve double coordination.

Alternatively, K_e^{cc} and K_r^{cc} can be set by formulating the problem as a general Nash-bargaining game problem (Nagarajan & Bassok, 2008; Palit & Brint, 2020).

Appendix C. Proofs of the analytical results

This appendix presents the proof of all analytical results stated in the manuscript.

C.1. Proof of Proposition 1

For w_e fixed, if $w_r = w_e$, we have a single player game and the buyer will acquire all needed units from the expedited supplier, that is, $Q_r = 0$ and $\Pi_r(w_e) = 0$. If $w_r = c_r$, $Q_r > 0$ but $\Pi_r(c_r) = 0$. Since $\Pi_r(c_r) = \Pi_r(w_e) = 0$ and since $\Pi_r(w_r) \geq 0$ for $c_r \leq w_r \leq w_e$ then, by Rolle's theorem, the function $\Pi_r(\cdot)$ attains a maximum between c_r and w_e. That is, the optimal price w_r^g satisfies $c_r < w_r^g < w_e$.

Now for w_r fixed, if $w_e \leq w_r$ then as mentioned above, we have a single player and $\Pi_e = (w_e - c_e)\mu$. While when $w_e > w_r$ both suppliers will be involved in the game and as w_e moves away from w_r, the regular quantity Q_r increases and the average profit $\Pi_e = (\mu - Q_r)(w_e - c_e)$ starts by decreasing, but since Q_r is bounded by μ the change in Q_r will start decreasing as Δw increases resulting in a change in Q_r small compared to a change in Δw. This in turn implies that Π_e will start increasing as Δw becomes big enough. Therefore, for the expedited supplier to achieve a maximum profit he has to choose between setting $w_e^g = w_r$ or setting $w_e^g = \bar{w}_e$ the maximum allowable value.

Remark.

To prove concavity of the profit (convexity of the cost), we note that since the profit is continuous in Q, concavity follows if we can show midpoint concavity that is $\frac{\Pi(Q_1, S^*_{Q_1}) + \Pi(Q_2, S^*_{Q_1})}{2} \leq \Pi(\frac{Q_1+Q_2}{2}, S^*_{\frac{Q_1+Q_2}{2}})$. This is achieved by showing that $\frac{\Pi(Q_1, S^*_{Q_1}) + \Pi(Q_2, S^*_{Q_2})}{2} \leq \Pi(\frac{Q_1+Q_2}{2}, E_1)$, where E_1 is the policy that orders from the emergency supplier in each period a quantity equals to the average of the quantities identified by the policy $S^*_{Q_1}$ and $S^*_{Q_2}$. Then the proof is complete since $\Pi(\frac{Q_1+Q_2}{2}, E_1) \leq \Pi(\frac{Q_1+Q_2}{2}, S^*_{\frac{Q_1+Q_2}{2}})$ because $S^*_{\frac{Q_1+Q_2}{2}}$ is the optimal policy when we order $\frac{Q_1+Q_2}{2}$ from the regular supplier.

C.2. Proof of Proposition 2

From the definition of Π^c_{Br} and because Q^c_r is its maximizer, one has

$$\begin{aligned}
\Pi^c_{Br}(Q^c_r) &= (p - w_e)\mu + (w_e - c_r)Q^c_r - hL_1(S^*_{Q^c_r}) - bL_2(S^*_{Q^c_r}) \\
&\quad - \int_0^\infty [(h+b)F_e(S^*_{Q^c_r} + x) - b][1 - H_{Q^c_r}(x)]dx \\
&\geq (p - w_e)\mu + (w_e - c_r)Q^g_r - hL_1(S^*_{Q^g_r}) - bL_2(S^*_{Q^g_r}) \\
&\quad - \int_0^\infty [(h+b)F_e(S^*_{Q^g_r} + x) - b][1 - H_{Q^g_r}(x)]dx \\
&= (p - w_e)\mu + (w_e - w_r)Q^g_r - hL_1(S^*_{Q^g_r}) - bL_2(S^*_{Q^g_r}) \\
&\quad - \int_0^\infty [(h+b)F_e(S^*_{Q^g_r} + x) - b][1 - H_{Q^g_r}(x)]dx \\
&\quad + (w_r - c_r)Q^g_r \\
&= \Pi^g_B(Q^g_r) + \Pi^g_r(Q^g_r).
\end{aligned}$$

Remark.

Regarding the equilibrium, for the expedited supplier to achieve a maximum profit, he has to choose between setting $w_e = w_r$ or setting $w_e = \bar{w}_e$, with $\bar{w}_e$ being the maximum allowable value. Anticipating this, the regular supplier will choose w_r as the largest value for which the expedited supplier will prefer $w_e = \bar{w}_e$. Such w_r^{gg} satisfies $(w_r^{gg} - c_e)\mu = (\bar{w}_e - c_e)(\mu - Q_r(w_r^{gg}, \bar{w}_e))$. Note that a solution to the previous equation always exits in $[c_e, \bar{w}_e]$ because the function $\eta(w_r) = (w_r - c_e)\mu - (\bar{w}_e - c_e)(\mu - Q_r(w_r, \bar{w}_e))$ is negative for $w_r = c_r$ and it is positive for w_r very close to $\bar{w}_e$.

C.3. Proof of Proposition 3

To establish point a), note that Eq. (6), gives

$$\frac{\partial \Pi^c_{Br}}{\partial Q^c_r} = w_e - c_r + \int_0^\infty [(h+b)F_e(S^*_{Q^c_r} + x) - b]\frac{\partial H_{Q^c_r}(x)}{\partial Q^c_r}dx = 0.$$

Taking the derivative of the above expression with respect to w_e yields

$$\frac{\partial^2 \Pi^c_{Br}}{\partial w_e \partial Q^c_r} = 1 + \frac{\partial^2 \Pi^c_{Br}}{\partial^2 Q^c_r}\frac{\partial Q^c_r}{\partial w_e} = 0,$$

which implies that $\frac{\partial Q^c_r}{\partial w_e} = -1/\frac{\partial^2 \Pi^c_{Br}}{\partial^2 Q^c_r} > 0$. The last inequality follows from the fact that Π^c_{Br} is concave in Q^c_r.

Next to prove (b), recall that the expected profit of the regular is $\Pi^g_r = (w^g_r - c_r)Q^g_r$. Taking derivative with respect to w_r gives

$$\frac{\partial \Pi^g_r}{\partial w^g_r} = Q^g_r + (w^g_r - c_r)\frac{\partial Q^g_r}{\partial w^g_r} = 0,$$

which implies that $\frac{\partial Q^g_r}{\partial w^g_r} = \frac{-Q^g_r}{w^g_r - c_r} < 0$. Moreover. differentiating $\frac{\partial \Pi^g_B}{\partial Q^g_r}$ with respect to w^g_r leads to

$$-1 + \frac{\partial^2 \Pi^g_B}{\partial^2 Q^g_r}\frac{\partial Q^g_r}{\partial w^g_r} = 0,$$

implying that $\frac{\partial^2 \Pi^g_B}{\partial^2 Q^g_r} = 1/\frac{\partial Q^g_r}{\partial w^g_r} = -\frac{w^g_r - c_r}{Q^g_r}$. Now, taking the derivative of $\frac{\partial \Pi^g_B}{\partial Q^g_r}$ with respect to w_e leads to:

$$1 - \frac{\partial w^g_r}{\partial w_e} + \frac{\partial^2 \Pi^g_B}{\partial^2 Q^g_r}\frac{\partial Q^g_r}{\partial w_e} = 0 \Longleftrightarrow 1 - \frac{\partial w^g_r}{\partial w_e} - \frac{w^g_r - c_r}{Q^g_r}\frac{\partial Q^g_r}{\partial w_e} = 0$$

$$\Rightarrow (1 - \frac{\partial w^g_r}{\partial w_e})Q^g_r - (w^g_r - c_r)\frac{\partial Q^g_r}{\partial w_e} = 0.$$

Consequently

$$\frac{\partial \Pi^g_r}{\partial w_e} = \frac{\partial w^g_r}{\partial w_e}Q^g_r + (w^g_r - c_e)\frac{\partial Q^g_r}{\partial w_e} = Q^g_r > 0.$$

To complete the proof, taking the derivative of Eq. (5) with respect to w_e leads to $\frac{\partial \Pi^c_{Br}}{\partial w_e} = Q^c_r - \mu < 0$. While, for $\Pi^c_{Br} - \Pi^g_r$ one sees that $\frac{\partial(\Pi^c_{Br} - \Pi^g_r)}{\partial w_e} = Q^c_r - \mu - Q^g_r < 0$.

C.4. Proof of Proposition 4

To establish (a) we just need to study the behavior of Q^c_r and Q^{cc}_r as functions of c_r. It is easy to see, that by taking the derivative of $\frac{\partial \Pi^c_{Br}}{\partial Q^c_r}$ with respect to c_r,

$$\frac{\partial^2 \Pi^c_{Br}}{\partial c_r \partial Q^c_r} = -1 + \frac{\partial^2 \Pi^c_{Br}}{\partial^2 Q^c_r}\frac{\partial Q^c_r}{\partial c_r} = 0 \Rightarrow \frac{\partial Q^c_r}{\partial c_r} = \frac{1}{\frac{\partial^2 \Pi^c_{Br}}{\partial^2 Q^c_r}} < 0.$$

The same argument is repeated for Q^{cc}_r. in fact

$$\frac{\partial^2 \Pi^{cc}}{\partial c_r \partial Q^{cc}_r} = -1 + \frac{\partial^2 \Pi^{cc}}{\partial^2 Q^{cc}_r}\frac{\partial Q^{cc}_r}{\partial c_r} = 0 \Rightarrow \frac{\partial Q^{cc}_r}{\partial c_r} = \frac{1}{\frac{\partial^2 \Pi^{cc}}{\partial^2 Q^{cc}_r}} < 0.$$

To prove (b) recall that $\Pi^c_e = (w_e - c_e)(\mu - Q^c_r)$, and observe that

$$\frac{\partial \Pi^c_e}{\partial c_r} = -(w_e - c_e)\frac{\partial Q^c_r}{\partial c_r} > 0.$$

Now for Π^g_r note that

$$\frac{\partial \Pi^g_B}{\partial Q^g_r} = w_e - w^g_r + \int_0^\infty [(h+b)F_e(S^*_{Q^g_r} + x) - b]\frac{\partial H_{Q^g_r}(x)}{\partial Q^g_r}dx = 0.$$

Differentiating the above with respect to c_r leads to $-\frac{\partial w^g_r}{\partial c_r} + \frac{\partial^2 \Pi^g_B}{\partial^2 Q^g_r}\frac{\partial Q^g_r}{\partial c_r} = 0$ or equivalently $-\frac{\partial w^g_r}{\partial c_r} - \frac{w^g_r - c_r}{Q^g_r}\frac{\partial Q^g_r}{\partial c_r} = 0$ implying that $Q^g_r\frac{\partial w^g_r}{\partial c_r} + (w^g_r - c_r)\frac{\partial Q^g_r}{\partial c_r} = 0$. Therefore, $\frac{\partial \Pi^g_r}{\partial c_r} = (\frac{\partial w^g_r}{\partial c_r} - 1)Q^g_r + (w^g_r - c_r)\frac{\partial Q^g_r}{\partial c_r} = -Q^g_r < 0$.

Next, for Π^c_{Br} direct computations show that $\frac{\partial \Pi^c_{Br}}{\partial c_r} = -Q^c_r < 0$. While for $\Pi^c_{Br} - \Pi^g_r$ one notes that $\frac{\partial(\Pi^c_{Br} - \Pi^g_r)}{\partial c_r} = -Q^c_r + Q^g_r < 0$ since $Q^g_r < Q^c_r$. Finally for Π^{cc}, one has $\frac{\partial \Pi^{cc}}{\partial c_r} = -Q^{cc}_r < 0$.

C.5. Proof of Proposition 5

To prove (a), one notices that Eq. (14) yields

$$\frac{\partial \Pi^{cc}}{\partial Q_r^{cc}} = c_e - c_r + \int_0^\infty [(h+b)F_e(S^*_{Q_r^{cc}} + x) - b]\frac{\partial H_{Q_r^{cc}}(x)}{\partial Q_r^{cc}}dx = 0,$$

which when differentiated with respect to c_e gives

$$\frac{\partial^2 \Pi^{cc}}{\partial c_e \partial Q_r^{cc}} = 1 + \frac{\partial^2 \Pi^{cc}}{\partial^2 Q_r^{cc}}\frac{\partial Q_r^{cc}}{\partial c_e} = 0 \Rightarrow \frac{\partial Q_r^{cc}}{\partial c_e} = \frac{-1}{\frac{\partial^2 \Pi^{cc}}{\partial^2 Q_r^{cc}}} > 0.$$

To establish (b), direct computations show that $\frac{\partial \Pi^{cc}}{\partial c_e} = Q_r^{cc} - \mu < 0$.

C.6. Proof of Proposition 6

Using the definition of Π^c_{Br} and Π^{cc} and taking derivative with respect to h shows that

$$\frac{\partial \Pi^c_{Br}}{\partial h} = -\int_0^\infty L_1(S^*_{Q_r^c} + x)dH_{Q_r^c(x)} < 0,$$

and

$$\frac{\partial \Pi^{cc}}{\partial h} = -\int_0^\infty L_1(S^*_{Q_r^{cc}} + x)dH_{Q_r^{cc}(x)} < 0.$$

C.7. Proof of Proposition 7

The steps are identical to Proposition 6 but taking derivative with respect to b instead of h. One gets

$$\frac{\partial \Pi^c_{Br}}{\partial b} = -\int_0^\infty L_2(S^*_{Q_r^c} + x)dH_{Q_r^c(x)} < 0,$$

and

$$\frac{\partial \Pi^{cc}}{\partial b} = -\int_0^\infty L_2(S^*_{Q_r^{cc}} + x)dH_{Q_r^{cc}(x)} < 0.$$

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